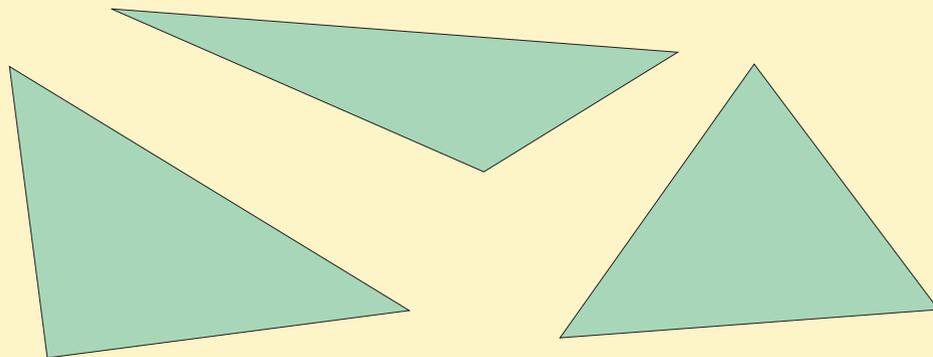


Materials:

- centimetre ruler
- 1-cm grid paper
- scissors

Work with a partner.



Part 1

- On grid paper, draw a large right triangle. Make sure its base is along a grid line and the third vertex is at a grid point. Estimate the area of the triangle.
- On another sheet of grid paper, draw a congruent triangle.
- Cut out both triangles. Place the triangles edge to edge to make a quadrilateral.
- Describe the quadrilateral. How many different quadrilaterals can you make? Sketch all your quadrilaterals.
- Calculate the area of each quadrilateral you made. Compare the area of each quadrilateral to the area of the triangle. What do you notice?

Part 2

- Draw a large acute triangle with its base along a grid line and the third vertex at a grid point. Estimate the area of the triangle.
- Draw a congruent triangle.
- Cut out both triangles. Then, cut along a grid line on each triangle to make 2 triangles.

- Arrange the 4 triangles edge to edge to make a quadrilateral with no gaps or overlaps.
- Describe the quadrilateral. How many different quadrilaterals can you make? Sketch all your quadrilaterals.
- Calculate the area of each quadrilateral you made. Calculate the area of the original acute triangle. Compare the areas. What do you notice?



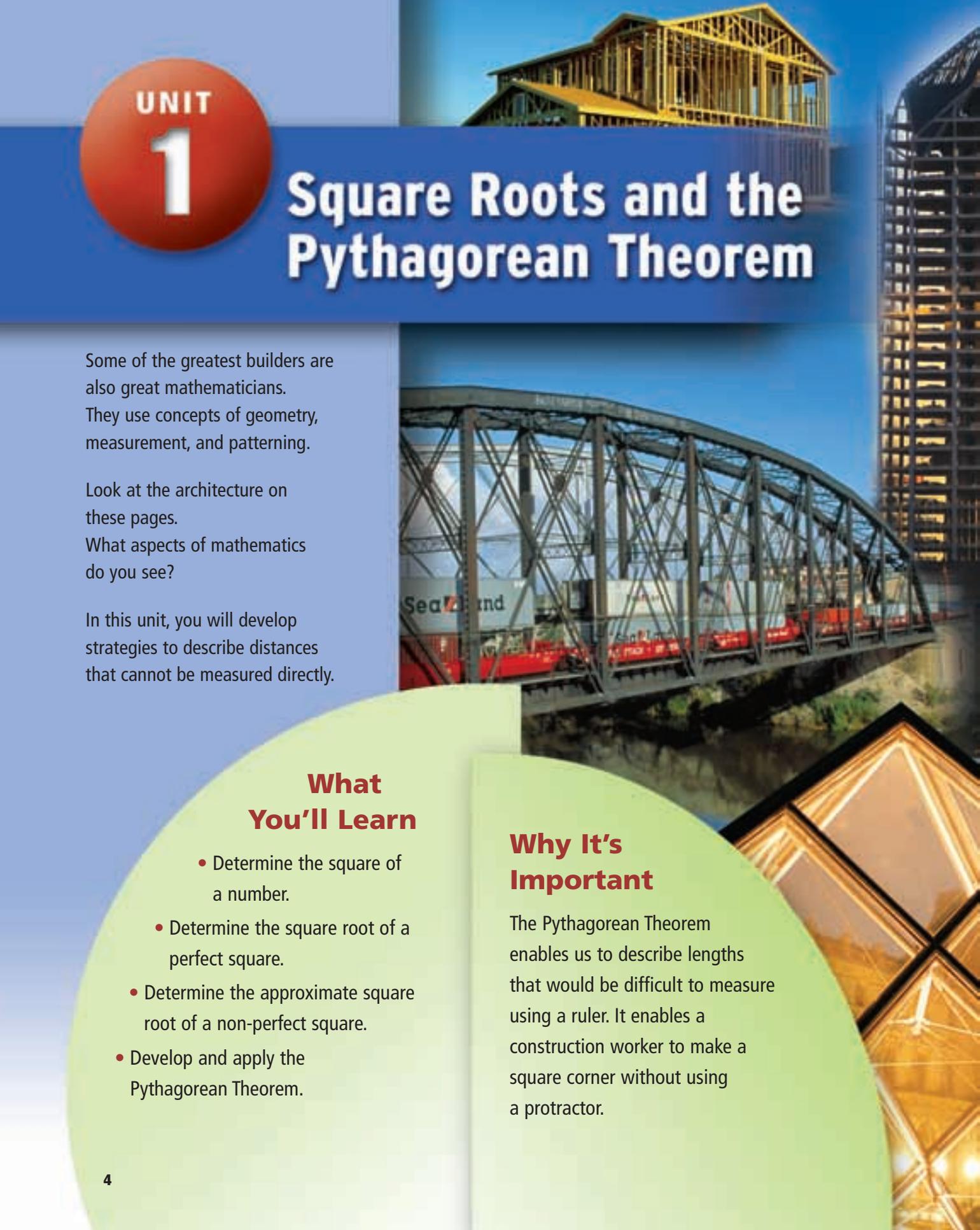
Part 3

Create a summary of your work.

Show all your calculations. Explain your thinking.

Take It Further

- Draw an obtuse triangle on grid paper. Predict its area.
- How did you use what you learned about acute and right triangles to make your prediction?
- Find a way to check your prediction.



UNIT

1

Square Roots and the Pythagorean Theorem

Some of the greatest builders are also great mathematicians. They use concepts of geometry, measurement, and patterning.

Look at the architecture on these pages.

What aspects of mathematics do you see?

In this unit, you will develop strategies to describe distances that cannot be measured directly.

What You'll Learn

- Determine the square of a number.
- Determine the square root of a perfect square.
- Determine the approximate square root of a non-perfect square.
- Develop and apply the Pythagorean Theorem.

Why It's Important

The Pythagorean Theorem enables us to describe lengths that would be difficult to measure using a ruler. It enables a construction worker to make a square corner without using a protractor.



Key Words

- square number
- perfect square
- square root
- leg
- hypotenuse
- Pythagorean Theorem
- Pythagorean triple

1.1

Square Numbers and Area Models

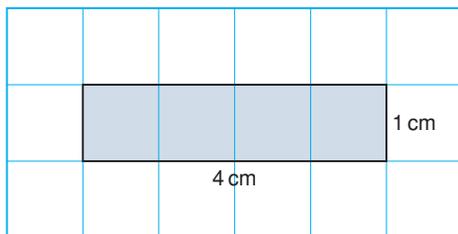
Focus

Relate the area of a square and square numbers.

A rectangle is a quadrilateral with 4 right angles.

A square also has 4 right angles.

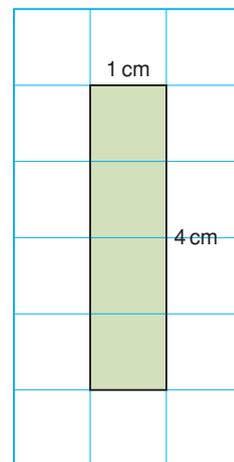
A rectangle with base 4 cm and height 1 cm is the same as a rectangle with base 1 cm and height 4 cm.



These two rectangles are *congruent*.

Is every square a rectangle?

Is every rectangle a square?



Investigate

Work with a partner.

You will need grid paper and 20 square tiles like this:

Use the tiles to make as many different rectangles as you can with each area.

4 square units

12 square units

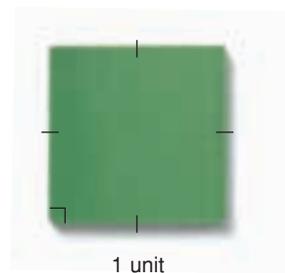
6 square units

16 square units

8 square units

20 square units

9 square units



Draw the rectangles on grid paper.

- For how many areas above were you able to make a square?
- What is the side length of each square you made?
- How is the side length of a square related to its area?

Reflect & Share

Compare your strategies and results with those of another pair of classmates.

Find two areas greater than 20 square units for which you could use tiles to make a square.

How do you know you could make a square for each of these areas?

Connect

When we multiply a number by itself, we *square* the number.

For example: The square of 4 is $4 \times 4 = 16$.

We write: $4 \times 4 = 4^2$

So, $4^2 = 4 \times 4 = 16$

We say: Four squared is sixteen.

16 is a **square number**, or a **perfect square**.

One way to model a square number is to draw a square whose area is equal to the square number.

Example 1

Show that 49 is a square number.

Use a diagram, symbols, and words.

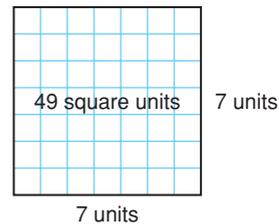
▶ A Solution

Draw a square with area 49 square units.

The side length of the square is 7 units.

Then, $49 = 7 \times 7 = 7^2$

We say: Forty-nine is seven squared.



Example 2

A square picture has area 169 cm^2 .

Find the perimeter of the picture.

▶ A Solution

The picture is a square with area 169 cm^2 .

Find the side length of the square:

Find a number which, when multiplied by itself, gives 169.

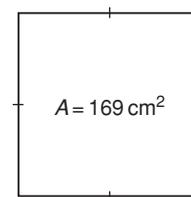
$$13 \times 13 = 169$$

So, the picture has side length 13 cm.

Perimeter is the distance around the picture.

$$\begin{aligned} \text{So, } P &= 13 \text{ cm} + 13 \text{ cm} + 13 \text{ cm} + 13 \text{ cm} \\ &= 52 \text{ cm} \end{aligned}$$

The perimeter of the picture is 52 cm.



Discuss the ideas

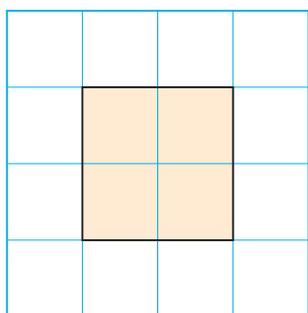
1. Is 1 a square number? How can you tell?
2. Suppose you know the area of a square. How can you find its perimeter?
3. Suppose you know the perimeter of a square. How can you find its area?

Practice

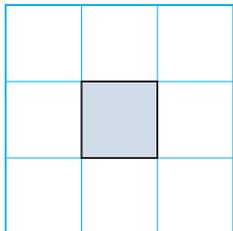
Check

4. Match each square below to its area.

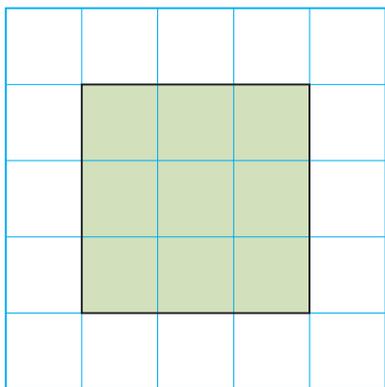
a)



b)



c)



- i) 1 unit \times 1 unit = 1 square unit
- ii) 2 units \times 2 units = 4 square units
- iii) 3 units \times 3 units = 9 square units

5. Find the area of a square with each side length.
a) 8 units b) 10 units c) 3 units

6. Use square tiles.
Make as many different rectangles as you can with area 36 square units.
Draw your rectangles on grid paper.
Is 36 a perfect square?
Justify your answer.

Apply

7. Use square tiles.
Make as many different rectangles as you can with area 28 square units.
Draw your rectangles on grid paper.
Is 28 a perfect square?
Justify your answer.



8. Show that 25 is a square number.
Use a diagram, symbols, and words.
9. Show that 12 is not a square number.
Use a diagram, symbols, and words.

10. Use a diagram to show that each number below is a square number.

- a) 1 b) 144
c) 121 d) 900

11. Find the side length of a square with each area.

- a) 100 m^2 b) 64 cm^2
c) 81 m^2 d) 400 cm^2

12. Which of these numbers is a perfect square?

How do you know?

- a) 10 b) 50
c) 81 d) 20

13. Use 1-cm grid paper.

Draw as many different rectangles as you can with area 64 cm^2 .

Find the base and height of each rectangle.

Record the results in a table.

Base (cm)	Height (cm)	Perimeter (cm)

Which rectangle has the least perimeter?

What can you say about this rectangle?

14. I am a square number.

The sum of my digits is 9.

What square numbers might I be?

15. These numbers are not square numbers. Which two consecutive square numbers is each number between?

Describe the strategy you used.

- a) 12 b) 40
c) 75 d) 200

16. The floor of a large square room has area 144 m^2 .

- a) Find the length of a side of the room.
b) How much baseboard is needed to go around the room?
c) Each piece of baseboard is 2.5 m long.

How many pieces of baseboard are needed?

What assumptions do you make?



17. A garden has area 400 m^2 .

The garden is divided into

16 congruent square plots.

Sketch a diagram of the garden.

What is the side length of each plot?

- 18. Assessment Focus** Which whole numbers between 50 and 200 are perfect squares?
Explain how you know.
- 19.** Lee is planning to fence a square kennel for her dog.
Its area must be less than 60 m^2 .
- Sketch a diagram of the kennel.
 - What is the kennel's greatest possible area?
 - Find the side length of the kennel.
 - How much fencing is needed?
 - One metre of fencing costs \$10.00. What is the cost of the fencing? What assumptions do you make?



- 20. Take It Further** Devon has a piece of poster board 45 cm by 20 cm. His teacher challenges him to cut the board into parts, then rearrange the parts to form a square.

- What is the side length of the square?
- What are the fewest cuts Devon could have made? Explain.

- 21. Take It Further** The digital root of a number is the result of adding the digits of the number until a single-digit number is reached. For example, to find the digital root of 147:

$$1 + 4 + 7 = 12 \text{ and } 1 + 2 = 3$$

- Find the digital roots of the first 15 square numbers. What do you notice?
- What can you say about the digital root of a square number?
- Use your results in part b. Which of these numbers might be square numbers?
 - 440
 - 2809
 - 3008
 - 4225
 - 625

Reflect

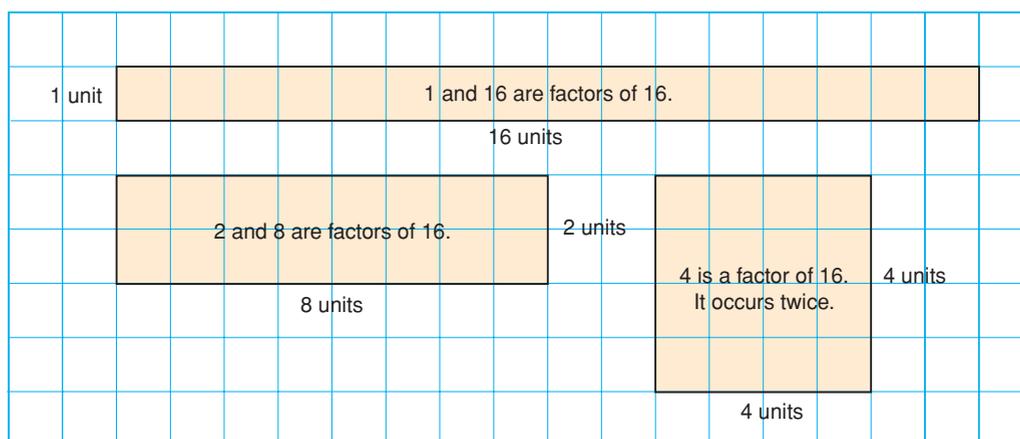
Use diagrams to explain why 24 is not a square number but 25 is a square number.

Connect

- Here are some ways to tell whether a number is a square number.
If we can find a division sentence for a number so that the quotient is equal to the divisor, the number is a square number.
For example, $16 \div 4 = 4$, so 16 is a square number.



- We can also use factoring.
Factors of a number occur in pairs.
These are the dimensions of a rectangle.



Sixteen has 5 factors: 1, 2, 4, 8, 16

Since there is an odd number of factors, one rectangle is a square.

The square has side length 4 units.

We say that 4 is a **square root** of 16.

We write: $4 = \sqrt{16}$

When a number has an odd number of factors, it is a square number.

When we multiply a number by itself, we square the number.

Squaring and taking the square root are inverse operations. That is, they undo each other.

$$4 \times 4 = 16$$

$$\text{so, } 4^2 = 16$$

$$\begin{aligned} \sqrt{16} &= \sqrt{4 \times 4} = \sqrt{4^2} \\ &= 4 \end{aligned}$$

A factor that occurs twice is only written once in the list of factors.

Example 1

Find the square of each number.

a) 5

b) 15

A Solution

a) The square of 5 is: $5^2 = 5 \times 5$
 $= 25$

b) The square of 15 is: $15^2 = 15 \times 15$
 $= 225$

Example 2

Find a square root of 64.

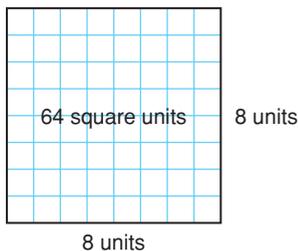
A Solution

Use grid paper.

Draw a square with area 64 square units.

The side length of the square is 8 units.

So, $\sqrt{64} = 8$



Example 2 Another Solution

Find pairs of factors of 64.

Use division facts.

$64 \div 1 = 64$ 1 and 64 are factors.

$64 \div 2 = 32$ 2 and 32 are factors.

$64 \div 4 = 16$ 4 and 16 are factors.

$64 \div 8 = 8$ 8 is a factor. It occurs twice.

The factors of 64 are: 1, 2, 4, **8**, 16, 32, 64

A square root of 64 is 8, the factor that occurs twice.

Example 3

The factors of 136 are listed in ascending order.

136: 1, 2, 4, 8, 17, 34, 68, 136

Is 136 a square number?

How do you know?

Numbers are in ascending order when they are written from least to greatest number.

A Solution

A square number has an odd number of factors.

One hundred thirty-six has 8 factors.

Eight is an even number.

So, 136 is not a square number.

Example 3 Another Solution

List the factors of 136 in a column, in ascending order.

Beside this column, list the factors in descending order.

Multiply the numbers in each row.

The same factor does not occur in the same place in both columns.

So, 136 cannot be written as the product of 2 equal numbers.

So, 136 is not a square number.

$$\begin{array}{r} 1 \times 136 = 136 \\ 2 \times 68 = 136 \\ 4 \times 34 = 136 \\ 8 \times 17 = 136 \\ 17 \times 8 = 136 \\ 34 \times 4 = 136 \\ 68 \times 2 = 136 \\ 136 \times 1 = 136 \end{array}$$

Discuss

the ideas

1. Squaring a number and taking a square root are inverse operations. What other inverse operations do you know?
2. When the factors of a perfect square are written in order from least to greatest, what do you notice?
3. Why do you think numbers such as 4, 9, 16, ... are called perfect squares?
4. Suppose you list the factors of a perfect square. Why is one factor a square root and not the other factors?

Practice

Check

- 5.** Find the square of each number.
- a) 4 b) 6
c) 2 d) 9
- 6.** Find.
- a) 8^2 b) 3^2
c) 1^2 d) 7^2
- 7.** Find a square root of each number.
- a) 25 b) 81
c) 64 d) 169
- 8.** a) Find the square of each number.
- i) 1 ii) 10
iii) 100 iv) 1000
- b) Use the patterns in part a. Predict the square of each number.
- i) 10 000 ii) 1 000 000
- 9.** a) Use a table like this.

Number = 50	
Factor Pairs	
1	50
2	

- List the factor pairs of each number. Which numbers are square numbers? How do you know?
- i) 50 ii) 100
iii) 144 iv) 85
- b) Find a square root of each square number in part a.

Apply

- 10.** List the factors of each number in ascending order.
Find a square root of each number.
- a) 256 b) 625 c) 121
- 11.** The factors of each number are listed in ascending order.
Which numbers are square numbers?
How do you know?
- a) 225: 1, 3, 5, 9, 15, 25, 45, 75, 225
b) 500: 1, 2, 4, 5, 10, 20, 25, 50, 100, 125, 250, 500
c) 324: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 81, 108, 162, 324
d) 160: 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160
- 12.** a) List the factors of each number in ascending order.
- i) 96 ii) 484
iii) 240 iv) 152
v) 441 vi) 54
- b) Which numbers in part a are square numbers?
How can you tell?
- 13.** Find each square root.
- a) $\sqrt{1}$ b) $\sqrt{49}$
c) $\sqrt{144}$ d) $\sqrt{9}$
e) $\sqrt{16}$ f) $\sqrt{100}$
g) $\sqrt{625}$ h) $\sqrt{225}$

14. Find a square root of each number.

- a) 3^2 b) 6^2
c) 10^2 d) 117^2

15. Find the square of each number.

- a) $\sqrt{4}$ b) $\sqrt{121}$
c) $\sqrt{225}$ d) $\sqrt{676}$

16. **Assessment Focus** Find each square root. Use a table, list, or diagram to support your answer.

- a) $\sqrt{169}$
b) $\sqrt{36}$
c) $\sqrt{196}$

17. Find the number whose square root is 23. Explain your strategy.

18. Use your results from questions 6b and 13d. Explain why squaring and taking the square root are inverse operations.

19. Order from least to greatest.

- a) $\sqrt{36}$, 36, 4, $\sqrt{9}$
b) $\sqrt{400}$, $\sqrt{100}$, 19, 15
c) $\sqrt{81}$, 81, $\sqrt{100}$, 11
d) $\sqrt{49}$, $\sqrt{64}$, $\sqrt{36}$, 9

20. Which perfect squares have square roots between 1 and 20?
How do you know?

21. **Take It Further**

a) Find the square root of each palindromic number.

A palindromic number is a number that reads the same forward and backward.

- i) $\sqrt{121}$
ii) $\sqrt{12\ 321}$
iii) $\sqrt{1\ 234\ 321}$
iv) $\sqrt{123\ 454\ 321}$

b) Continue the pattern.
Write the next 4 palindromic numbers in the pattern and their square roots.

22. **Take It Further**

- a) Find.
i) 2^2 ii) 3^2
iii) 4^2 iv) 5^2
- b) Use the results from part a.
Find each sum.
i) $2^2 + 3^2$ ii) $3^2 + 4^2$
iii) $2^2 + 4^2$ iv) $3^2 + 5^2$
- c) Which sums in part b are square numbers?
What can you say about the sum of two square numbers?

Reflect

Which method of determining square numbers are you most comfortable with? Justify your choice.

1.3

Measuring Line Segments

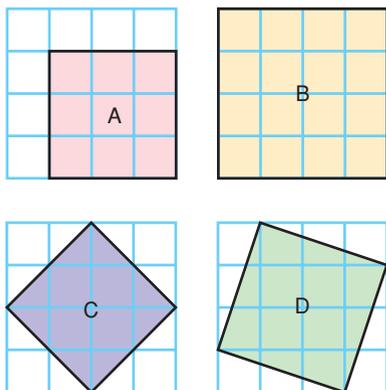
Focus Use the area of a square to find the length of a line segment.

Investigate

Work with a partner. You will need 1-cm grid paper.

Copy the squares below.

Without using a ruler, find the area and side length of each square.



What other squares can you draw on a 4 by 4 grid?

Find the area and side length of each square.

Write all your measurements in a table.

Reflect & Share

How many squares did you draw?

Describe any patterns in your measurements.

How did you find the area and side length of each square?

How did you write the side lengths of squares C and D?

Connect

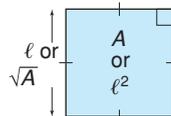
We can use the properties of a square to find its area or side length.

Area of a square = length \times length = (length)²

When the side length is ℓ , the area is ℓ^2 .

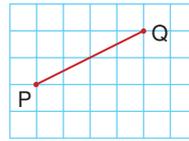
When the area is A , the side length is \sqrt{A} .

We can calculate the length of any line segment on a grid by thinking of it as the side length of a square.



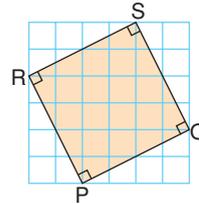
Example 1

Find the length of line segment PQ.

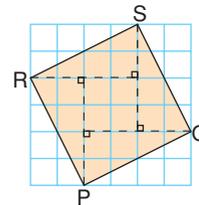


A Solution

Use a straightedge and protractor to construct a square on line segment PQ. Then, the length of the line segment is the square root of the area.



Cut the square into 4 congruent triangles and a smaller square.



The area of each triangle is: $A = \frac{bh}{2}$

Substitute: $b = 4$ and $h = 2$

$$\begin{aligned} A &= \frac{4 \times 2}{2} \\ &= 4 \end{aligned}$$

The area of each triangle is 4 square units.

The area of the 4 triangles is:

$$4 \times 4 \text{ square units} = 16 \text{ square units}$$

The area of the small square is: $A = \ell^2$

Substitute: $\ell = 2$

$$\begin{aligned} A &= 2^2 \\ &= 4 \end{aligned}$$

The area of the small square is: 4 square units

The area of PQRS = Area of triangles + Area of small square

$$= 16 \text{ square units} + 4 \text{ square units}$$

$$= 20 \text{ square units}$$

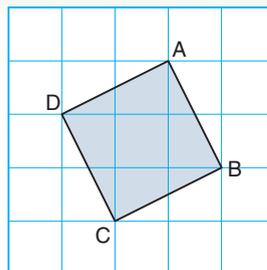
So, the side length of the square, $PQ = \sqrt{20}$ units

Since 20 is not a square number, we cannot write $\sqrt{20}$ as a whole number. In Lesson 1.4, you will learn how to find an approximate value for $\sqrt{20}$ as a decimal.

Example 2 illustrates another way to find the area of a square with its vertices where grid lines meet.

Example 2

- Find the area of square ABCD.
- What is the side length AB of the square?



A Solution

- Draw an enclosing square JKLM.
The area of JKLM = 3^2 square units
= 9 square units

The triangles formed by the enclosing square are congruent.

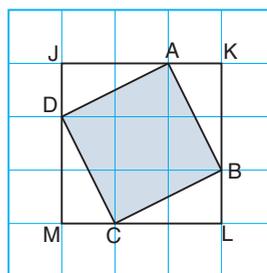
Each triangle has area:

$$\frac{1}{2} \times 1 \text{ unit} \times 2 \text{ units} = 1 \text{ square unit}$$

So, the 4 triangles have area:

$$4 \times 1 \text{ square unit} = 4 \text{ square units}$$

$$\begin{aligned} \text{The area of ABCD} &= \text{Area of JKLM} - \text{Area of triangles} \\ &= 9 \text{ square units} - 4 \text{ square units} \\ &= 5 \text{ square units} \end{aligned}$$



- So, the side length $AB = \sqrt{5}$ units

Discuss

the ideas

- Is the area of every square a square number?
- When will the length of a line segment not be a whole number?

Practice

Check

3. Simplify.

- a) 3^2 b) 4^2 c) 7^2
 d) 10^2 e) 6^2 f) 12^2

4. Find.

- a) $\sqrt{1}$ b) $\sqrt{64}$ c) $\sqrt{144}$
 d) $\sqrt{169}$ e) $\sqrt{121}$ f) $\sqrt{625}$

5. The area A of a square is given.

Find its side length.

Which side lengths are whole numbers?

- a) $A = 36 \text{ cm}^2$ b) $A = 49 \text{ m}^2$
 c) $A = 95 \text{ cm}^2$ d) $A = 108 \text{ m}^2$

6. The side length s of a square is given.

Find its area.

- a) $s = 8 \text{ cm}$ b) $s = \sqrt{44} \text{ cm}$
 c) $s = \sqrt{7} \text{ m}$ d) $s = 13 \text{ m}$

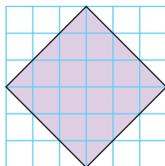
Apply

7. Copy each square on grid paper.

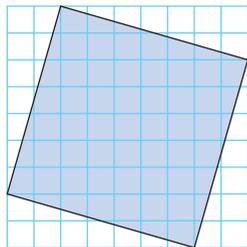
Find its area.

Then write the side length of the square.

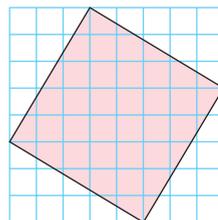
a)



b)



c)

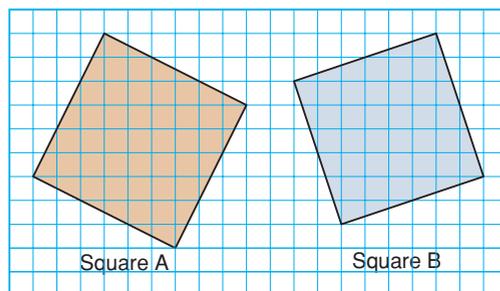


8. Copy each square on grid paper.

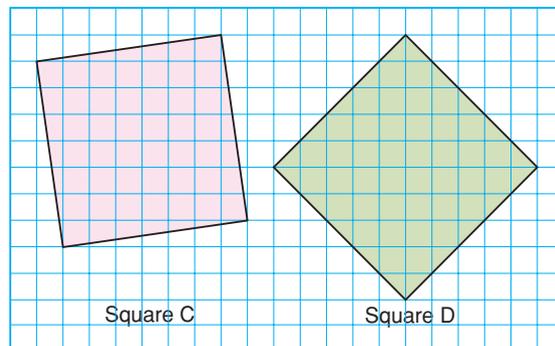
Which square in each pair has the greater area?

Show your work.

a)



b)

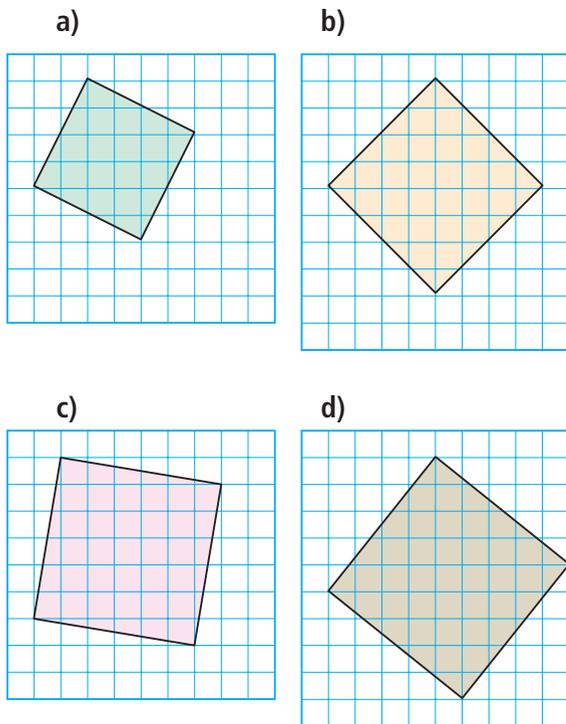


9. Copy each square on grid paper.

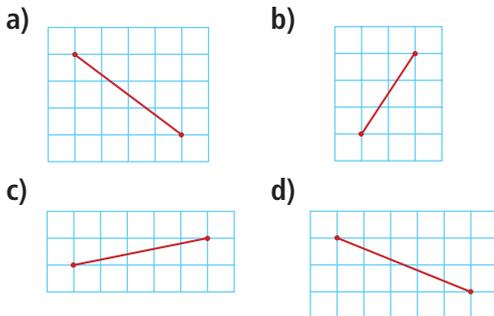
Find its area.

Order the squares from least to greatest area.

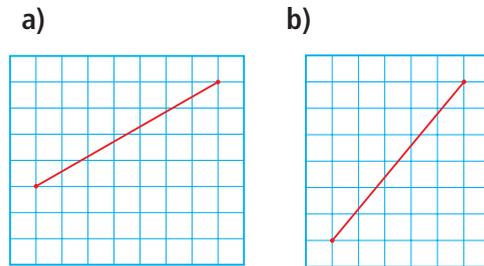
Then write the side length of each square.



- 10.** Copy each line segment on grid paper. Draw a square on each line segment. Find the area of the square and the length of the line segment.



- 11. Assessment Focus** Without measuring, determine which line segment is shorter. Explain how you know.



- 12. Take It Further** Plot each pair of points on a coordinate grid. Join the points to form a line segment. Find the length of the line segment.

- a) $P(1, 3), Q(5, 5)$
 b) $R(-3, 2), S(1, -3)$
 c) $T(-4, 2), U(-1, 3)$
 d) $W(6, 0), X(8, 2)$

- 13. Take It Further** On square dot paper, draw a square with area 2 square units. Write to explain how you know the square has this area.

- 14. Take It Further** Use a 7-cm by 7-cm grid. Construct a square with side length 5 cm. No side can be horizontal or vertical. Explain your strategy.

Reflect

How is the area of a square related to its side length?
 How can we use this relationship to find the length of a line segment?
 Include an example in your explanation.

1.4

Estimating Square Roots

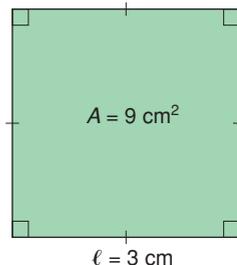
Focus Develop strategies for estimating a square root.

You know that a square root of a given number is a number which, when multiplied by itself, results in the given number.

$$\begin{aligned}\text{For example, } \sqrt{9} &= \sqrt{3 \times 3} \\ &= 3\end{aligned}$$

You also know that the square root of a number is the side length of a square with area that is equal to that number.

$$\text{For example, } \sqrt{9} = 3$$



Investigate

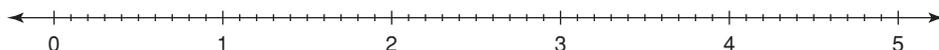
Work with a partner.

Use a copy of the number line below.

Place each square root on the number line to show its approximate value: $\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$, $\sqrt{18}$, $\sqrt{24}$

Write each estimated square root as a decimal.

Use grid paper if it helps.



Reflect & Share

Compare your answers with those of another pair of classmates.

What strategies did you use to estimate the square roots?

How could you use a calculator to check your estimates?



Connect

Here is one way to estimate the value of $\sqrt{20}$:

- 25 is the square number closest to 20, but greater than 20.

On grid paper, draw a square with area 25.

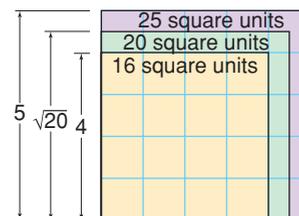
Its side length is: $\sqrt{25} = 5$

- 16 is the square number closest to 20, but less than 20.

Draw a square with area 16.

Its side length is: $\sqrt{16} = 4$

Draw the squares so they overlap.



A square with area 20 lies between these two squares.

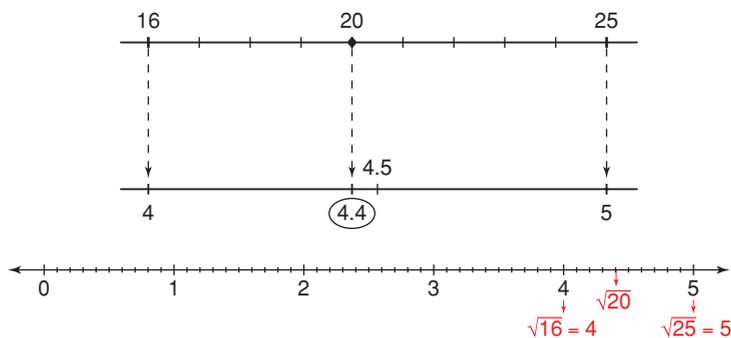
Its side length is $\sqrt{20}$.

20 is between 16 and 25, but closer to 16.

So, $\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$, but closer to $\sqrt{16}$.

So, $\sqrt{20}$ is between 4 and 5, but closer to 4.

An estimate of $\sqrt{20}$ is 4.4 to one decimal place.



Example 1

Which whole number is $\sqrt{96}$ closer to?

How do you know?

▶ A Solution

$$81 < 96 < 100$$

$$\text{So, } \sqrt{81} < \sqrt{96} < \sqrt{100}$$

$$9 < \sqrt{96} < 10$$

$\sqrt{96}$ is between 9 and 10.

96 is closer to 100 than to 81.

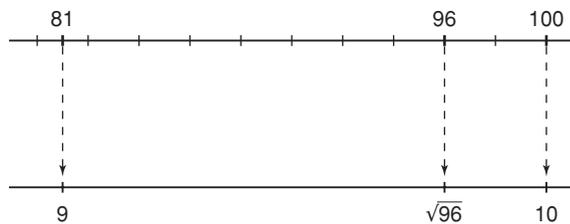
So, $\sqrt{96}$ is closer to $\sqrt{100}$, or 10.

▶ **Example 1**
Another Solution

Use number lines.

96 is between 81 and 100, but closer to 100.

So, $\sqrt{96}$ is between $\sqrt{81}$ and $\sqrt{100}$,
but closer to $\sqrt{100}$, or 10.



Example 2

A square garden has area 139 m^2 .

- a) What are the approximate dimensions of the garden to two decimal places?
- b) Net-wire fencing is needed to keep out coyotes. About how much fencing would be needed around the garden?

▶ **A Solution**

- a) Draw a square to represent the garden.

The side length of the square is: $\sqrt{139}$

Estimate:

$$121 < 139 < 144$$

$$\text{So, } 11 < \sqrt{139} < 12$$

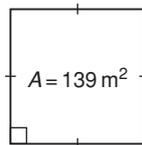
With a calculator, use guess and test to refine the estimate.

Try 11.5: $11.5 \times 11.5 = 132.25$ (too small)

Try 11.8: $11.8 \times 11.8 = 139.24$ (too large, but close)

Try 11.78: $11.78 \times 11.78 = 138.7684$ (close)

Try 11.79: $11.79 \times 11.79 = 139.0041$ (very close)



The side length of the garden is 11.79 m, to two decimal places.

- b) To find how much fencing is needed, find the perimeter of the garden.

The perimeter of the garden is about:

$$4 \times 11.79 \text{ m} = 47.16 \text{ m}$$

To be sure there is enough fencing, round up.

About 48 m of fencing are needed to go around the garden.

Discuss the ideas

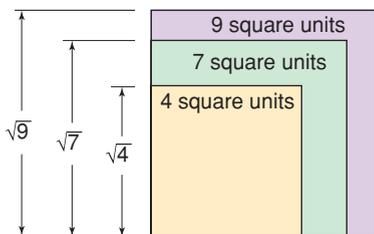
1. Which type of number has an exact square root?
2. Which type of number has an approximate square root?
3. How can you use perfect squares to estimate a square root, such as $\sqrt{8}$?

Practice

Check

4. Find.
- a) $\sqrt{15 \times 15}$
 - b) $\sqrt{22 \times 22}$
 - c) $\sqrt{3 \times 3}$
 - d) $\sqrt{1 \times 1}$
5. Between which two consecutive whole numbers is each square root? How do you know?
- a) $\sqrt{5}$
 - b) $\sqrt{11}$
 - c) $\sqrt{57}$
 - d) $\sqrt{38}$
 - e) $\sqrt{171}$
 - f) $\sqrt{115}$

6. Copy this diagram on grid paper. Then estimate the value of $\sqrt{7}$ to one decimal place.



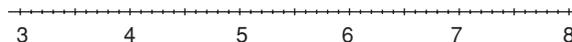
7. Use the number line below.
- a) Which placements are good estimates of the square roots? Explain your reasoning.



- b) Use the number line to estimate the value of each square root that is incorrectly placed.

Apply

8. Use a copy of this number line. Place each square root on the number line to show its approximate value.
- a) $\sqrt{11}$
 - b) $\sqrt{40}$
 - c) $\sqrt{30}$
 - d) $\sqrt{55}$



9. In each pair, is the given whole number greater than, less than, or equal to the square root? Justify your answer.
- a) 7, $\sqrt{14}$
 - b) 8, $\sqrt{60}$
 - c) 11, $\sqrt{121}$
 - d) 12, $\sqrt{150}$

10. Which whole number is each square root closer to? How do you know?
- a) $\sqrt{58}$
 - b) $\sqrt{70}$
 - c) $\sqrt{90}$
 - d) $\sqrt{151}$

11. Is each statement true or false? Explain.

- a) $\sqrt{17}$ is between 16 and 18.
- b) $\sqrt{5} + \sqrt{5}$ is equal to $\sqrt{10}$.
- c) $\sqrt{131}$ is between 11 and 12.



12. Use guess and test to estimate each square root to two decimal places.

Record each trial.

- a) $\sqrt{23}$ b) $\sqrt{13}$ c) $\sqrt{78}$
- d) $\sqrt{135}$ e) $\sqrt{62}$ f) $\sqrt{45}$



13. Find the approximate side length of the square with each area.

Give each answer to one decimal place.

- a) 92 cm^2 b) 430 m^2
- c) 150 cm^2 d) 29 m^2

14. Which estimates are good estimates of the square roots?

Explain your reasoning.

- a) $\sqrt{17}$ is about 8.50.
- b) $\sqrt{20}$ is about 4.30.
- c) $\sqrt{8}$ is about 2.83.
- d) $\sqrt{34}$ is about 5.83.



15. **Assessment Focus** A student uses a square canvas for her painting. The canvas has area 5 m^2 . She wants to frame her artwork.

- a) What are the dimensions of the square frame to two decimal places?
- b) The framing can be purchased in 5-m or 10-m lengths. Which length of framing should she purchase? Justify your choice.



16. A square lawn is to be reseeded. The lawn has area 152 m^2 .

- a) What are the approximate dimensions of the lawn to two decimal places?
- b) A barrier of yellow tape is placed around the lawn to keep people off. About how much tape is needed?



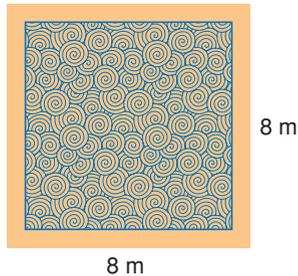
17. Which is the closer estimate of $\sqrt{54}$: 7.34 or 7.35? How did you find out?



18. Most classrooms are rectangles. Measure the dimensions of your classroom. Calculate its area. Suppose your classroom was a square with the same area. What would its dimensions be?

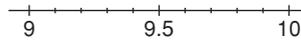


- 19. Take It Further** A square carpet covers 75% of the area of a floor. The floor is 8 m by 8 m.



- a) What are the dimensions of the carpet?
Give your answer to two decimal places.
- b) What area of the floor is not covered by the carpet?
- 20. Take It Further** Is the product of two perfect squares always, sometimes, or never a perfect square? Investigate to find out. Write about your findings.
- 21. Take It Further** An approximate square root of a whole number is 7.67. Is the whole number closer to 49 or 64? How do you know?

- 22. Take It Further** Write five numbers whose square roots are between 9 and 10. Explain your strategy.



- 23. Take It Further** Simplify each expression. Give your answer to two decimal places when necessary.

- a) $\sqrt{81} + \sqrt{16}$
 b) $\sqrt{81 + 16}$
 c) $\sqrt{\sqrt{81} + 16}$
 d) $\sqrt{81 + \sqrt{16}}$
 e) $\sqrt{\sqrt{81} + \sqrt{16}}$



- 24. a)** Estimate each square root to two decimal places.
 i) $\sqrt{2}$ ii) $\sqrt{200}$ iii) $\sqrt{20\,000}$
 b) Look at your results in part a. What patterns do you see?
 c) Use the patterns in part b to estimate.

Reflect

What is your favourite method for estimating a square root of a number that is not a perfect square?
Use an example to explain how you would use your method.



Fitting In

HOW TO PLAY

Your teacher will give you 3 sheets of game cards.
Cut out the 54 cards.

1. Place the 1, 5, and 9 cards on the table.

Spread them out so there is room for several cards between them.

Shuffle the remaining cards.

Give each player 6 cards.

2. All cards laid on the table must be arranged from least to greatest.

Take turns to place a card so it touches another card on the table.

- It can be placed to the right of the card if its value is greater.
- It can be placed to the left of the card if its value is less.
- It can be placed on top of the card if its value is equal.
- However, it cannot be placed between two cards that are already touching.

YOU WILL NEED

1 set of *Fitting In* game cards; scissors

NUMBER OF PLAYERS

2 to 4

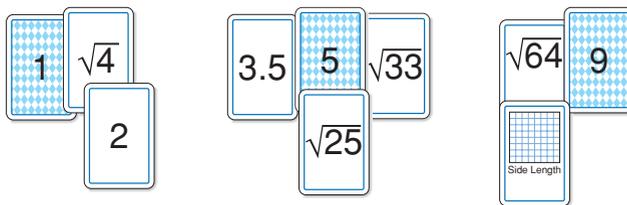
GOAL OF THE GAME

To get the lowest score

In this example, the $\sqrt{16}$ card cannot be placed because the 3.5 and the 5 cards are touching.

The player cannot play that card in this round.

What other games could you play with these cards? Try out your ideas.



3. Place as many of your cards as you can. When no player can place any more cards, the round is over.

Your score is the number of cards left in your hand.

At the end of five rounds, the player with the lowest score wins.

Investigating Square Roots with a Calculator



Focus Use a calculator to investigate square roots.

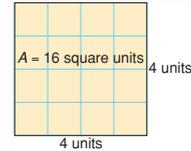


We can use a calculator to calculate a square root.

- Use the square root key to find the square root of 16.

A square root of 16 is 4.

Check by multiplying or by squaring: $4 \times 4 = 16$



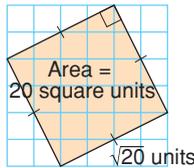
- Many square roots are not whole numbers.

Use a calculator to find a square root of 20.

4.472135955 should be displayed.

A square root of 20 is 4.5 to one decimal place.

Investigate to compare what happens when we use a calculator with a $\sqrt{\quad}$ key versus a 4-function calculator.



- Find $\sqrt{20} \times \sqrt{20}$ using each calculator.

Record what you see in the display each time.

Which display is accurate? How do you know?

- Check what happens when you enter $4.472135955 \times 4.472135955$ into each calculator. Suppose you multiplied using pencil and paper. Would you expect a whole number or a decimal? Explain.

- $\sqrt{20}$ cannot be described exactly by a decimal.

The decimal for $\sqrt{20}$ never repeats and never terminates.

We can write $\sqrt{20}$ with different levels of accuracy.

For example, $\sqrt{20}$ is 4.5 to one decimal place, 4.47 to two decimal places, and 4.472 to three decimal places.

The calculator rounds or truncates the decimal for $\sqrt{20}$. So, the number in the display is only an approximate square root.

Check

Find each square root.

Which square roots are approximate?

Justify your answer.

a) $\sqrt{441}$

b) $\sqrt{19}$

c) $\sqrt{63}$

d) $\sqrt{529}$

Mid-Unit Review

LESSON

1.1

1. Which numbers below are perfect squares? Draw diagrams to support your answers.

a) 15 b) 26 c) 65 d) 100

1.2

2. Find a square root of each number.

a) 16 b) 49 c) 196 d) 400

3. Find.

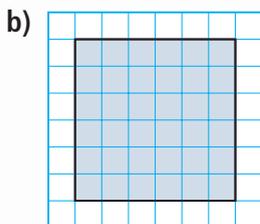
a) 11^2 b) $\sqrt{64}$ c) $\sqrt{169}$ d) $\sqrt{225}$

1.1

1.2

4. Copy each square onto 1-cm grid paper.

- i) Find the area of each square.
ii) Write the side length of each square as a square root.



5. List the factors of each number below in order from least to greatest.

Which of the numbers are square numbers? How do you know?

For each square number below, write a square root.

a) 216 b) 364 c) 729

1.3

6. If you know a square number, how can you find its square root? Use diagrams, symbols, and words.

7. a) The area of a square is 24 cm^2 . What is its side length?

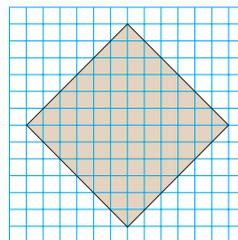
Why is the side length not a whole number?

- b) The side length of a square is 9 cm. What is its area?

1.3

1.4

8. Copy this square onto 1-cm grid paper.



- a) What is the area of the square?
b) Write the side length of the square as a square root.
c) Estimate the side length to one decimal place.

9. Find.

a) $\sqrt{12 \times 12}$ b) $\sqrt{34 \times 34}$

10. Between which two consecutive whole numbers does each square root lie?

How do you know? Sketch a number line to show your answers.

a) $\sqrt{3}$ b) $\sqrt{65}$ c) $\sqrt{72}$ d) $\sqrt{50}$



11. Use guess and test to estimate each square root to two decimal places. Record each trial.

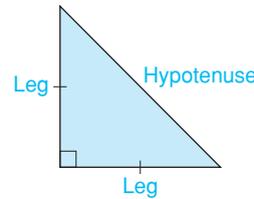
a) $\sqrt{17}$ b) $\sqrt{108}$ c) $\sqrt{33}$ d) $\sqrt{79}$

1.5

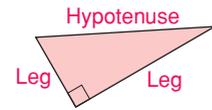
The Pythagorean Theorem

Focus Discover a relationship among the side lengths of a right triangle.

We can use the properties of a right triangle to find the length of a line segment. A right triangle has two sides that form the right angle. The third side of the right triangle is called the **hypotenuse**. The two shorter sides are called the **legs**.



Isosceles right triangle



Scalene right triangle

Investigate

Work on your own.

You will need grid paper, centimetre cubes, and a protractor.



- Copy line segment AB.
Draw right triangle ABC that has segment AB as its hypotenuse.
Draw a square on each side of $\triangle ABC$.
Find the area and side length of each square.
- Draw 3 different right triangles, with a square on each side.
Find the area and side length of each square.
Record your results in a table.

See page 18 if you have forgotten how to find the area of a square on a line segment.

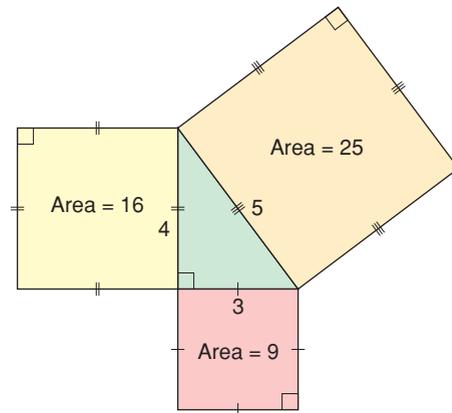
	Area of Square on Leg 1	Length of Leg 1	Area of Square on Leg 2	Length of Leg 2	Area of Square on Hypotenuse	Length of Hypotenuse
Triangle ABC						
Triangle 1						
Triangle 2						
Triangle 3						

Reflect & Share

Compare your results with those of another classmate. What relationship do you see among the areas of the squares on the sides of a right triangle? How could this relationship help you find the length of a side of a right triangle?

Connect

Here is a right triangle, with a square drawn on each side.



The area of the square on the hypotenuse is 25.
The areas of the squares on the legs are 9 and 16.

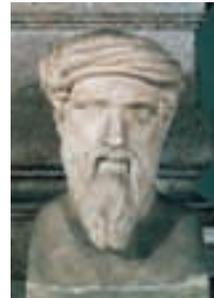
Notice that: $25 = 9 + 16$

A similar relationship is true for all right triangles.

In a right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

This relationship is called the **Pythagorean Theorem**.

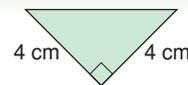
We can use this relationship to find the length of any side of a right triangle, when we know the lengths of the other two sides.



The Pythagorean Theorem is named for the Greek mathematician, Pythagoras.

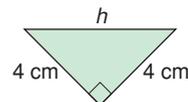
Example 1

Find the length of the hypotenuse.
Give the length to one decimal place.



▶ A Solution

Label the hypotenuse h .
The area of the square on the hypotenuse is h^2 .
The areas of the squares on the legs are
 $4 \times 4 = 16$ and $4 \times 4 = 16$.
So, $h^2 = 16 + 16$
 $= 32$



The area of the square on the hypotenuse is 32.

So, the side length of the square is: $h = \sqrt{32}$

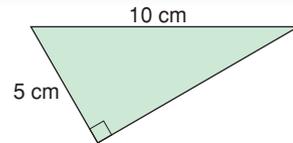
Use a calculator.

$$h \doteq 5.6569$$

So, the hypotenuse is 5.7 cm to one decimal place.

Example 2

Find the unknown length to one decimal place.



A Solution

Label the leg g .

The area of the square on the hypotenuse is $10 \times 10 = 100$.

The areas of the squares on the legs are g^2 and $5 \times 5 = 25$.

$$\text{So, } 100 = g^2 + 25$$

To solve this equation, subtract 25 from each side.

$$100 - 25 = g^2 + 25 - 25$$

$$75 = g^2$$

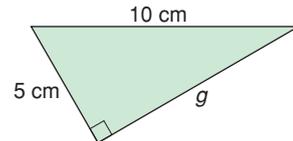
The area of the square on the leg is 75.

So, the side length of the square is: $g = \sqrt{75}$

Use a calculator.

$$g \doteq 8.66025$$

So, the leg is 8.7 cm to one decimal place.



Discuss the ideas

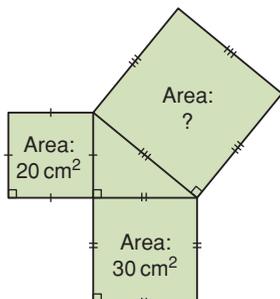
1. How can you identify the hypotenuse of a right triangle?
2. How can you use the Pythagorean Theorem to find the length of the diagonal in a rectangle?

Practice

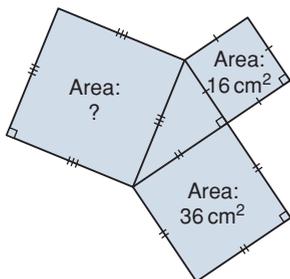
Check

3. Find the area of the indicated square.

a)

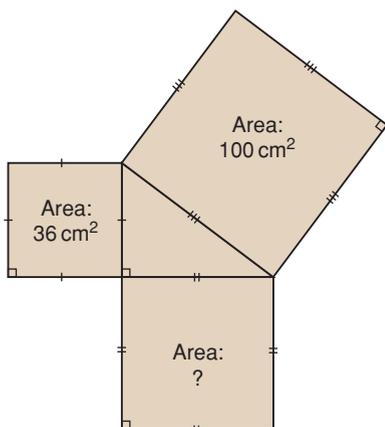


b)

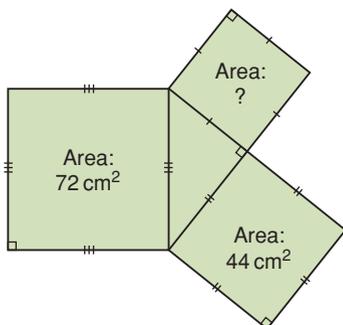


4. Find the area of the indicated square.

a)



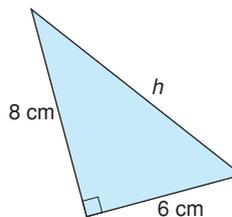
b)



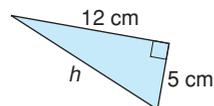
5. Find the length of each hypotenuse.

Give your answers to one decimal place where needed.

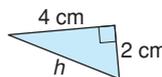
a)



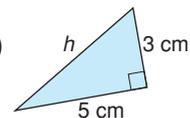
b)



c)



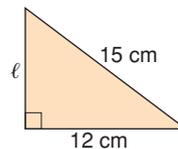
d)



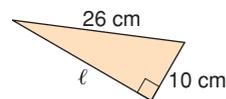
6. Find the length of each leg labelled ℓ .

Give your answers to one decimal place where needed.

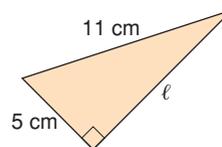
a)



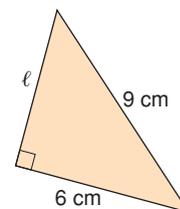
b)



c)



d)

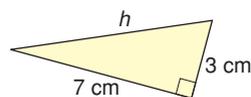


Apply

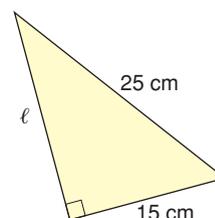
7. Find the length of each side labelled with a variable.

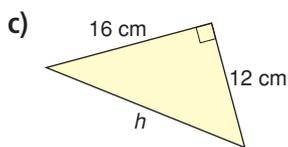
Give your answers to one decimal place where needed.

a)

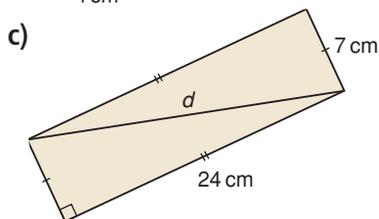
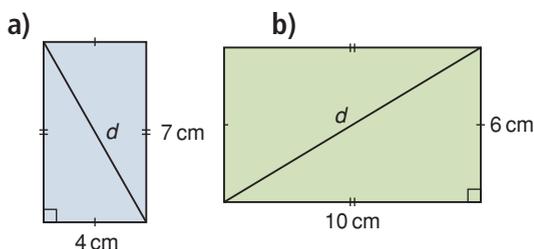


b)

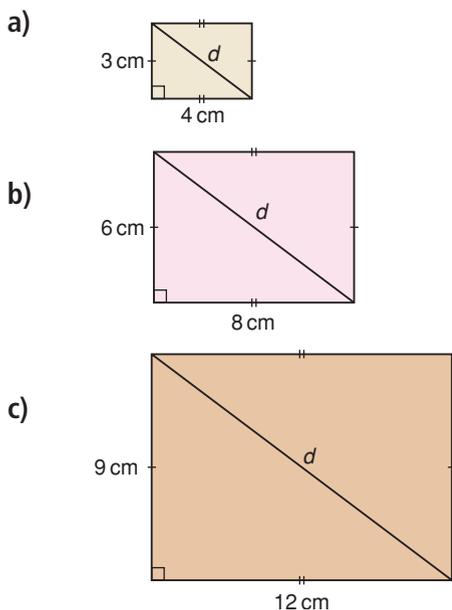




8. Find the length of the diagonal, d , in each rectangle.
Give your answers to two decimal places where needed.

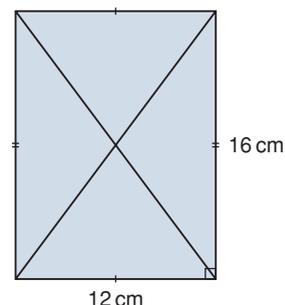


9. Find the length of the diagonal, d , in each rectangle.
What patterns do you notice?
Write to explain.
Use your patterns to draw the next rectangle in the pattern.



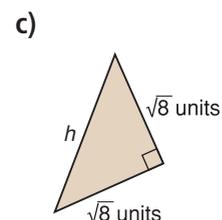
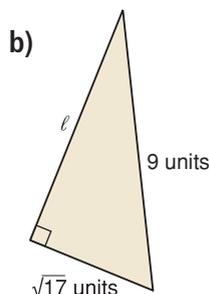
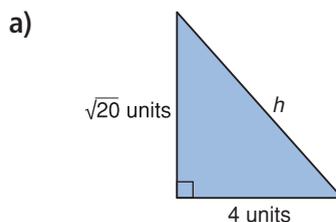
10. Suppose you are given the side lengths of a right triangle.
Which length is the length of the hypotenuse?
Explain how you know.

11. Use the rectangle on the right. Explain why the diagonals of a rectangle have the same length.



12. **Assessment Focus** The hypotenuse of a right triangle is $\sqrt{18}$ units.
What are the lengths of the legs of the triangle?
How many different answers can you find?
Sketch a triangle for each answer.
Explain your strategies.

13. Find the length of each side labelled with a variable.

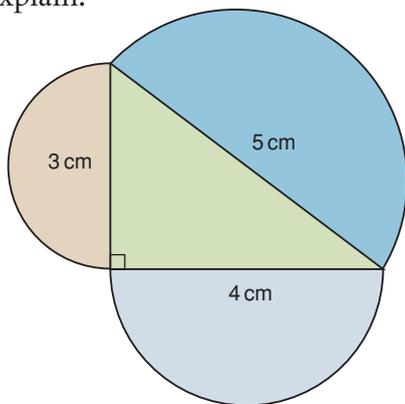


14. Use what you know about the Pythagorean Theorem. On grid paper, draw a line segment with each length. Explain how you did it.

- a) $\sqrt{5}$ b) $\sqrt{10}$
 c) $\sqrt{13}$ d) $\sqrt{17}$

15. Take It Further The length of the hypotenuse of a right triangle is 15 cm. The lengths of the legs are whole numbers of centimetres. Find the sum of the areas of the squares on the legs of the triangle. What are the lengths of the legs? Show your work.

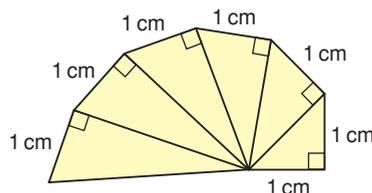
16. Take It Further An artist designed this logo. It is a right triangle with a semicircle drawn on each side of the triangle. Calculate the area of each semicircle. What do you notice? Explain.



17. Take It Further Use grid paper. Draw a right triangle with a hypotenuse with each length.

- a) $\sqrt{20}$ units b) $\sqrt{89}$ units c) $\sqrt{52}$ units

18. Take It Further Your teacher will give you a copy of part of the Wheel of Theodorus.



Verifying the Pythagorean Theorem



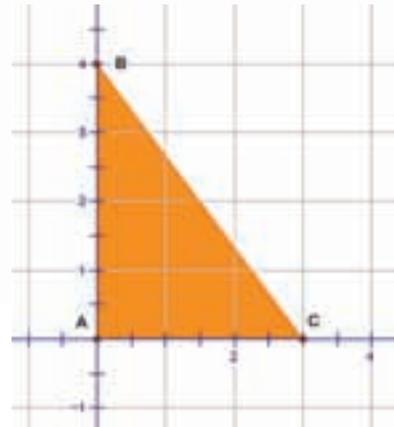
Focus Use a computer to investigate the Pythagorean relationship.

Geometry software can be used to create and transform shapes. Use available geometry software.

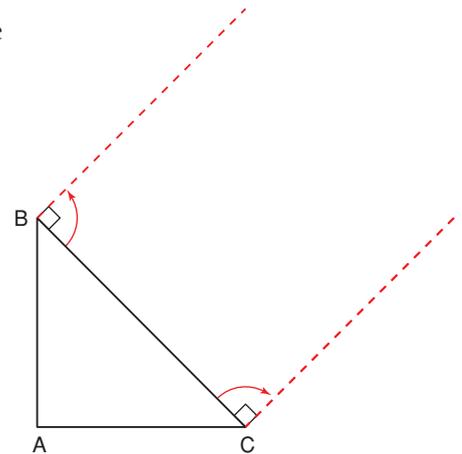
Open a new sketch.
Display a coordinate grid.

Should you need help at any time, use the software's Help menu.

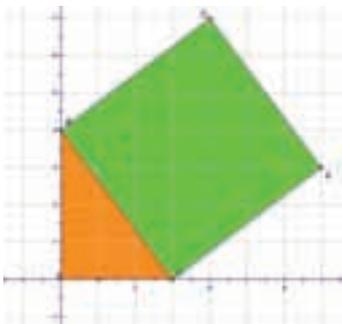
- Construct points $A(0, 0)$, $B(0, 4)$ and $C(3, 0)$. Join the points to form right $\triangle ABC$.



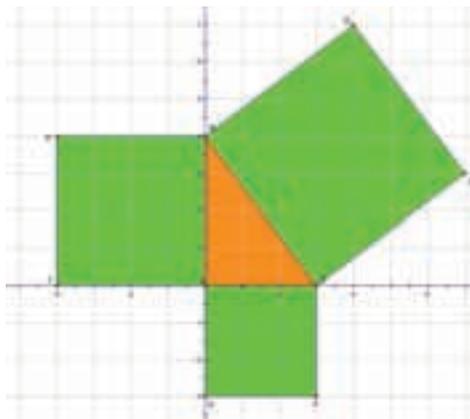
- Construct a square on each side of the triangle.
To form a square on BC, rotate line segment BC 90° clockwise about C, then 90° counterclockwise about B.



- Join the points at the ends of the rotated segments to form a square. Label the vertices of the square. This is what you should see on the screen.



- Repeat the previous step, with appropriate rotations, to form squares on AB and AC.



- Use the software to find the area of each square. What relationship is shown?
- Drag a vertex of the triangle and observe what happens to the area measurements of the squares. How does the geometry software verify the Pythagorean Theorem?

Use the software to investigate “what if” questions.

- What if the triangle was an acute triangle? Is the relationship still true? Explain.
- What if the triangle was an obtuse triangle? Is the relationship still true? Explain.



1.6

Exploring the Pythagorean Theorem

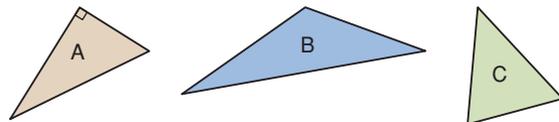
Focus Use the Pythagorean Theorem to identify right triangles.

Look at these triangles.

Which triangle is a right triangle? How do you know?

Which triangle is an obtuse triangle? An acute triangle?

How did you decide?



Investigate

Work on your own.

You will need grid paper and a protractor.

- Draw an obtuse triangle.
Draw a square on each side.
Find the area and side length of each square.
- Draw an acute triangle.
Draw a square on each side.
Find the area and side length of each square.
- Record your results in a table.

	Area of Square on Shortest Side	Length of Shortest Side	Area of Square on Second Side	Length of Second Side	Area of Square on Longest Side	Length of Longest Side
Obtuse Triangle						
Acute Triangle						

Reflect & Share

Compare your results with those of 3 other classmates.

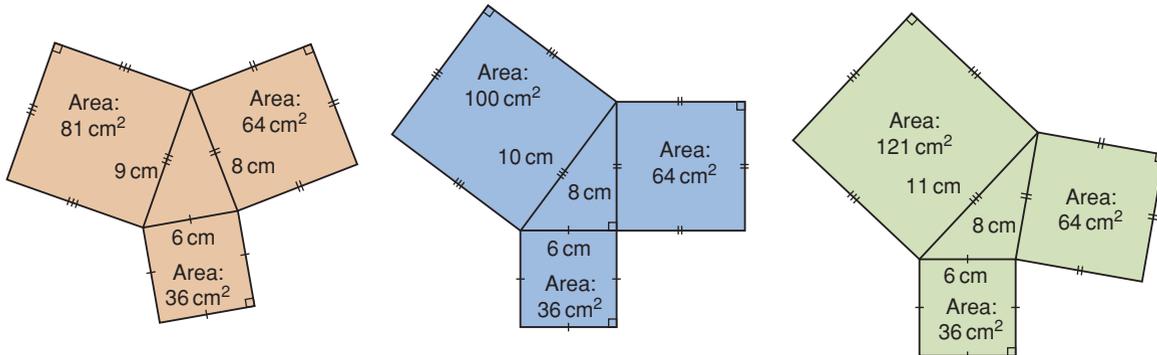
What relationship do you see among the areas of the squares on the sides of an obtuse triangle?

What relationship do you see among the areas of the squares on the sides of an acute triangle?

How could these relationships help you identify the type of triangle when you know the lengths of its sides?

Connect

Here are an acute triangle, a right triangle, and an obtuse triangle, with squares drawn on the sides of each triangle.



For each triangle, compare the sum of the areas of the two smaller squares to the area of the largest square.

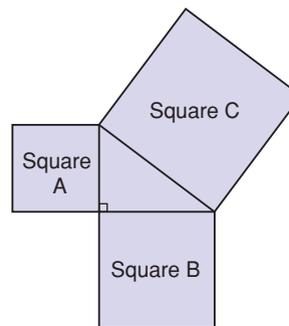
Triangle	Sum of Areas of Two Smaller Squares	Area of Largest Square
Acute	$36 + 64 = 100$	81
Right	$36 + 64 = 100$	100
Obtuse	$36 + 64 = 100$	121

Notice that the Pythagorean Theorem is true for the right triangle only.

We can use these results to identify whether a triangle is a right triangle.

If Area of square A + Area of square B = Area of square C,
then the triangle is a right triangle.

If Area of square A + Area of square B \neq Area of square C,
then the triangle is not a right triangle.



The symbol \neq means "does not equal."

Example 1

Determine whether each triangle with the given side lengths is a right triangle.

- a) 6 cm, 6 cm, 9 cm b) 7 cm, 24 cm, 25 cm

A Solution

- a) Sketch a triangle with a square on each side.

The square with the longest side is square C.

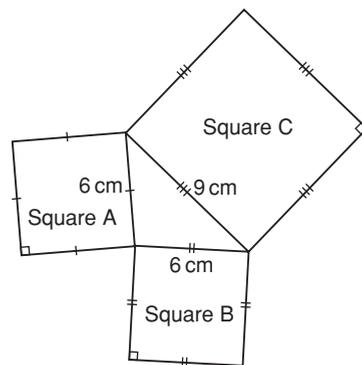
$$\begin{aligned}\text{Area of square A} &= 6 \text{ cm} \times 6 \text{ cm} \\ &= 36 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of square B} &= 6 \text{ cm} \times 6 \text{ cm} \\ &= 36 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of square C} &= 9 \text{ cm} \times 9 \text{ cm} \\ &= 81 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of square A} + \text{Area of square B} &= 36 \text{ cm}^2 + 36 \text{ cm}^2 \\ &= 72 \text{ cm}^2\end{aligned}$$

Since $72 \text{ cm}^2 \neq 81 \text{ cm}^2$, the triangle is not a right triangle.



- b) Sketch a triangle with a square on each side.

The square with the longest side is square C.

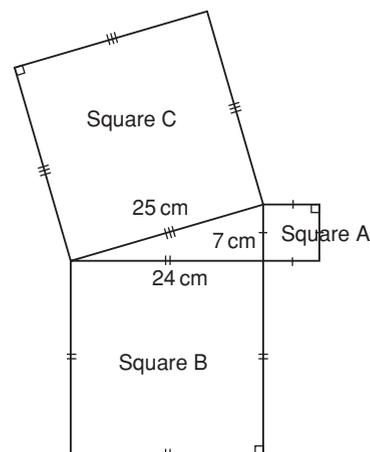
$$\begin{aligned}\text{Area of square A} &= 7 \text{ cm} \times 7 \text{ cm} \\ &= 49 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of square B} &= 24 \text{ cm} \times 24 \text{ cm} \\ &= 576 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of square C} &= 25 \text{ cm} \times 25 \text{ cm} \\ &= 625 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of square A} + \text{Area of square B} &= 49 \text{ cm}^2 + 576 \text{ cm}^2 \\ &= 625 \text{ cm}^2\end{aligned}$$

Since $625 \text{ cm}^2 = 625 \text{ cm}^2$, the triangle is a right triangle.



A set of 3 whole numbers that satisfies the Pythagorean Theorem is called a **Pythagorean triple**.

For example, 3-4-5 is a Pythagorean triple because $3^2 + 4^2 = 5^2$.

From part b of *Example 1*, a triangle with side lengths 7 cm, 24 cm, and 25 cm is a right triangle.

So, 7-24-25 is a Pythagorean triple.

Example 2

Which of these sets of numbers is a Pythagorean triple?

How do you know?

a) 8, 15, 18

b) 11, 60, 61

▶ A Solution

Suppose each set of numbers represents the side lengths of a triangle.

When the set of numbers satisfies the Pythagorean Theorem,

the set is a Pythagorean triple.

a) Check:

Does $8^2 + 15^2 = 18^2$?

$$\text{L.S.} = 8^2 + 15^2$$

$$= 64 + 225$$

$$= 289$$

$$\text{R.S.} = 18^2$$

$$= 324$$

Since $289 \neq 324$, 8-15-18 is not a Pythagorean triple.

b) Check:

Does $11^2 + 60^2 = 61^2$?

$$\text{L.S.} = 11^2 + 60^2$$

$$= 121 + 3600$$

$$= 3721$$

$$\text{R.S.} = 61^2$$

$$= 3721$$

Since $3721 = 3721$, 11-60-61 is a Pythagorean triple.

Discuss

the ideas

1. Suppose a square has been drawn on each side of a triangle. How can you tell whether the triangle is obtuse? How can you tell whether the triangle is acute?
2. Can a right triangle have a hypotenuse of length $\sqrt{65}$ units?

Math Link

History

"Numbers Rule the Universe!" That was the belief held by a group of mathematicians called the Brotherhood of Pythagoreans. Their power and influence became so strong that fearful politicians forced them to disband. Nevertheless, they continued to meet in secret and teach Pythagoras' ideas.

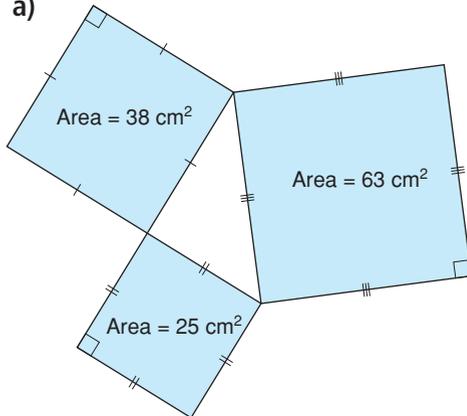


Practice

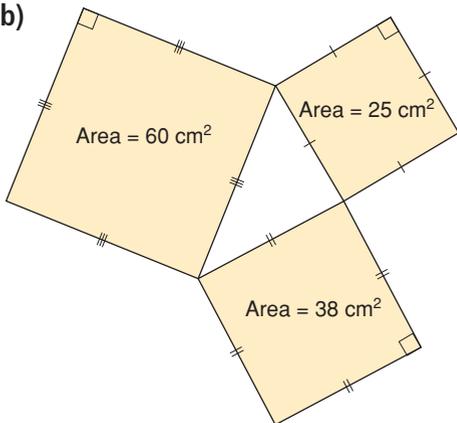
Check

3. The area of the square on each side of a triangle is given. Is the triangle a right triangle? How do you know?

a)

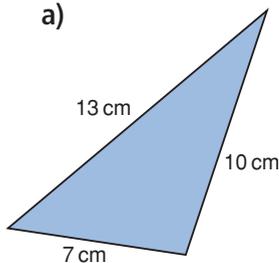


b)

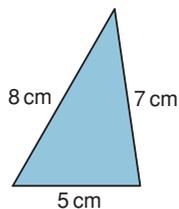


4. Which of these triangles appears to be a right triangle? Determine whether each triangle is a right triangle. Justify your answers.

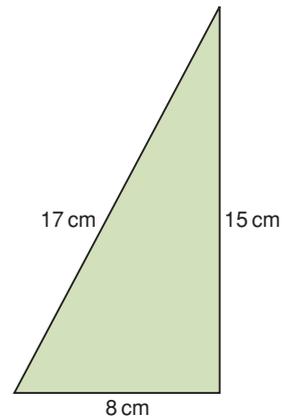
a)



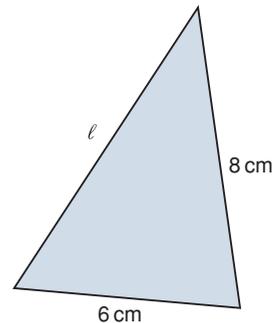
b)



c)



5. Look at the triangle below. Can the Pythagorean Theorem be used to find the length of the side labelled with a variable? Why or why not?



Apply

6. Determine whether a triangle with each set of side lengths is a right triangle. Justify your answers.
- 16 cm, 30 cm, 34 cm
 - 8 cm, 10 cm, 12 cm
 - 20 m, 25 m, 15 m
 - 28 m, 53 m, 45 m
 - 17 mm, 14 mm, 5 mm
 - 30 mm, 9 mm, 25 mm
 - 9 cm, 9 cm, 15 cm
 - 10 cm, 26 cm, 24 cm

7. Which sets of numbers below are Pythagorean triples?

How did you decide?

- a) 16, 30, 34
 - b) 6, 8, 9
 - c) 15, 39, 36
 - d) 16, 65, 63
 - e) 9, 30, 35
 - f) 40, 42, 58
8. An elder and his granddaughter, Kashala, are laying a plywood floor in a cabin.

The floor is rectangular, with side lengths 9 m and 12 m.

Kashala measures the diagonal of the floor as 15 m.

Is the angle between the two sides a right angle?

Justify your answer.



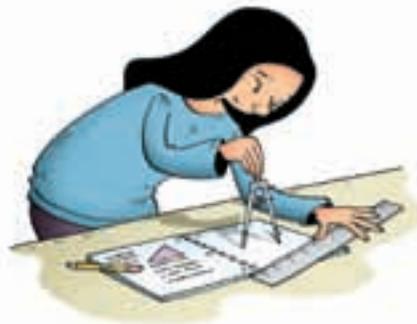
9. A triangle has side lengths 6 cm, 7 cm, and $\sqrt{13}$ cm.

Is this triangle a right triangle?

Do these side lengths form a Pythagorean triple? Explain.

10. Assessment Focus

May Lin uses a ruler and compass to construct a triangle with side lengths 3 cm, 5 cm, and 7 cm. Before May Lin constructs the triangle, how can she tell if the triangle will be a right triangle? Explain.



11. Look at the Pythagorean triples below.

3, 4, 5 6, 8, 10
12, 16, 20 15, 20, 25
21, 28, 35

- a) Each set of numbers represents the side lengths of a right triangle. What are the lengths of the legs? What is the length of the hypotenuse?
- b) Describe any pattern you see in the Pythagorean triples.
- c) Use a pattern similar to the one you found in part b. Generate 4 more Pythagorean triples from the triple 5, 12, 13.

12. Two numbers in a Pythagorean triple are given.

Find the third number.

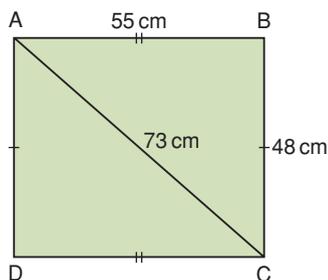
How did you find out?

- a) 14, 48,
- b) 32, 24,
- c) 12, 37,
- d) 20, 101,

- 13.** In Ancient Egypt, the Nile River overflowed every year and destroyed property boundaries. Because the land plots were rectangular, the Egyptians needed a way to mark a right angle. The Egyptians tied 12 evenly spaced knots along a piece of rope and made a triangle. Explain how this rope could have been used to mark a right angle.



- 14.** Is quadrilateral ABCD a rectangle? Justify your answer.



- 15. Take It Further** The perimeter of a right triangle is 90 m. The length of the longest side of the triangle is 41 m. Find the lengths of the other two sides. How did you find out?

- 16. Take It Further** Use your data and those of your classmates from *Investigate*, on page 39.

- Use the results for the obtuse triangles. How does the area of the square on the longest side compare to the sum of the areas of the squares on the other two sides?
- Use the results for the acute triangles. How does the area of the square on the longest side compare to the sum of the areas of the squares on the other two sides?
- Use the patterns in parts a and b to classify all the triangles in question 6.

- 17. Take It Further** You can use expressions to generate the numbers in a Pythagorean triple. Choose a number, then choose a greater number. Use these expressions to find the numbers in a Pythagorean triple:

- $2(\text{lesser number})(\text{greater number})$
- $(\text{greater number})^2 - (\text{lesser number})^2$
- $(\text{greater number})^2 + (\text{lesser number})^2$

If a spreadsheet is available, enter the formulas. Change the numbers you start with to generate 15 Pythagorean triples.

Reflect

When you know the side lengths of a triangle, how can you tell whether it is a right triangle?

What other condition must be satisfied for the numbers to be a Pythagorean triple?

Use examples in your explanation.

1.7

Applying the Pythagorean Theorem

Focus Solve problems using the Pythagorean Theorem.

Investigate

Work with a partner to solve this problem.

A doorway is 2.0 m high and 1.0 m wide.
 A square piece of plywood has side length 2.2 m.
 Can the plywood fit through the door?
 How do you know?
 Show your work.



Reflect & Share

Compare your solution with that of another pair of classmates.
 If the solutions are different, find out which solution is correct.
 What strategies did you use to solve the problem?

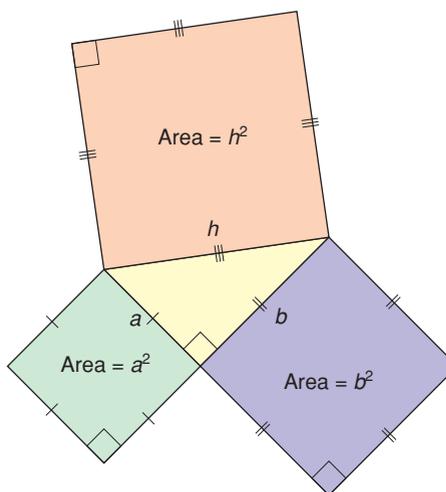
Connect

Since the Pythagorean Theorem is true for all right triangles, we can use an algebraic equation to describe it.

In the triangle at the right, the hypotenuse has length h , and the legs have lengths a and b .

The area of the square on the hypotenuse is $h \times h$, or h^2 .

The areas of the squares on the legs are $a \times a$ and $b \times b$, or a^2 and b^2 .



So, we can say: $h^2 = a^2 + b^2$

When we use this equation, remember that the lengths of the legs are represented by a and b , and the length of the hypotenuse by h .

We can use the Pythagorean Theorem to solve problems that involve right triangles.

Example 1

Marina helped her dad build a small rectangular table for her bedroom. The tabletop has length 56 cm and width 33 cm. The diagonal of the tabletop measures 60 cm. Does the tabletop have square corners? How do you know?

A square corner is a right angle.

A Solution

Draw a rectangle to represent the tabletop.

Suppose the diagonal of the tabletop is the hypotenuse, h , of a right triangle.

One leg is 33 cm.

The other leg is 56 cm.

Use the Pythagorean Theorem to find h .

$$h^2 = a^2 + b^2$$

Substitute: $a = 33$ and $b = 56$

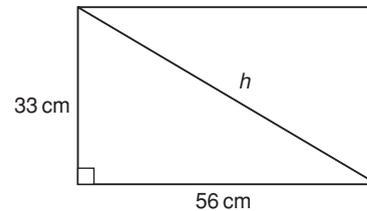
$$\begin{aligned} h^2 &= 33^2 + 56^2 \\ &= 1089 + 3136 \\ &= 4225 \end{aligned}$$

$$\begin{aligned} \text{So, } h &= \sqrt{4225} \\ &= 65 \end{aligned}$$

For the triangle to be a right triangle, the diagonal should measure 65 cm.

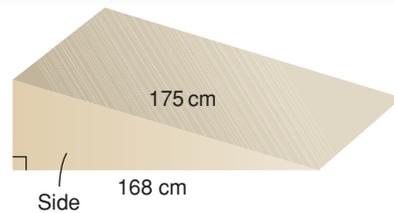
It measures 60 cm.

So, the tabletop does not have square corners.



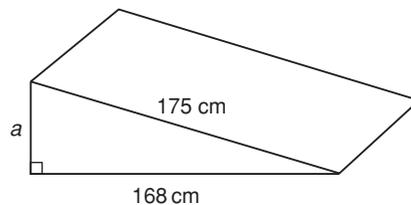
Example 2

A ramp is used to load a snow machine onto a trailer.
The ramp has horizontal length 168 cm
and sloping length 175 cm.
The side view is a right triangle.
How high is the ramp?



▶ A Solution

The side face of the ramp is a right triangle with hypotenuse 175 cm.
One leg is 168 cm.
The other leg is the height.
Label it a .



Use the Pythagorean Theorem.

$$h^2 = a^2 + b^2$$

Substitute: $h = 175$ and $b = 168$

$$175^2 = a^2 + 168^2$$

Use a calculator.

$$30\,625 = a^2 + 28\,224$$

Subtract 28 224 from each side to isolate a^2 .

$$30\,625 - 28\,224 = a^2 + 28\,224 - 28\,224$$

$$2401 = a^2$$

The area of the square with side length a is 2401 cm^2 .

$$a = \sqrt{2401}$$

Use a calculator.

$$= 49$$

The ramp is 49 cm high.

Discuss

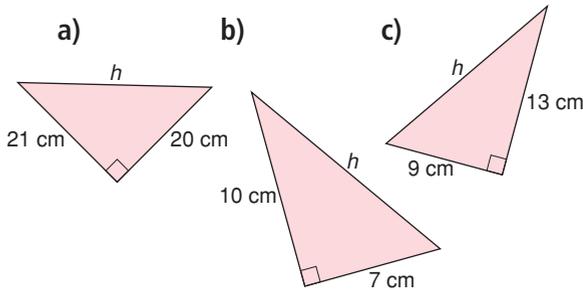
the ideas

1. What do you need to know to apply the Pythagorean Theorem to calculate a distance?
2. Does it matter which two sides of the triangle you know?
3. When you use the Pythagorean Theorem, how can you tell whether to add or subtract the areas?

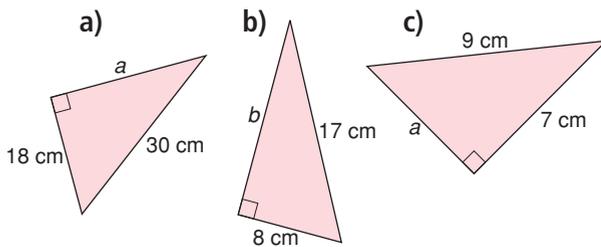
Practice

Check

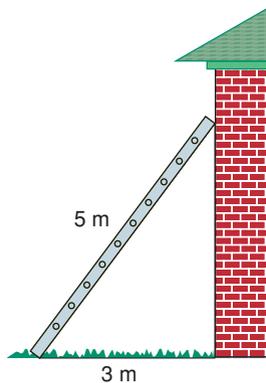
4. Find the length of each hypotenuse. Give your answers to one decimal place where needed.



5. Find the length of each leg labelled with a variable. Give your answers to one decimal place where needed.



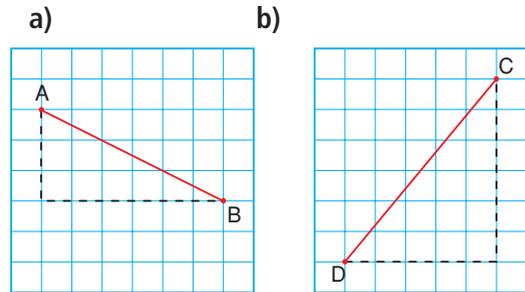
6. A 5-m ladder leans against a house. It is 3 m from the base of the wall. How high does the ladder reach?



Apply

7. As part of a design for a book cover, Brandon constructed a right triangle with sides 10 cm and 24 cm.
- How long is the third side?
 - Why are two answers possible to part a?

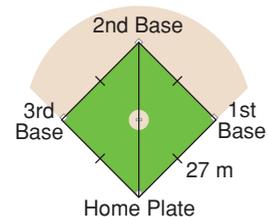
8. Find the length of each line segment. Give your answers to one decimal place.



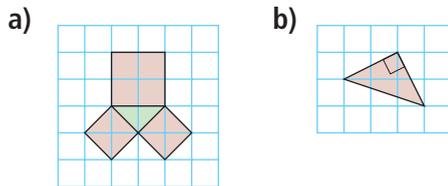
9. Alyssa has made a picture frame for the painting she just finished. The frame is 60 cm long and 25 cm wide. To check that the frame has square corners, Alyssa measures a diagonal. How long should the diagonal be? Sketch a diagram to illustrate your answer.



10. A boat is 35 m due south of a dock. Another boat is 84 m due east of the dock. How far apart are the boats?
11. A baseball diamond is a square with side length about 27 m. The player throws the ball from second base to home plate. How far did the player throw the ball? Give your answer to two decimal places.



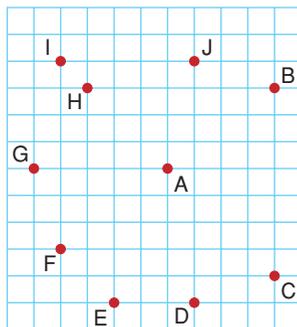
- 12.** Copy each diagram on grid paper. Explain how each diagram can be used to illustrate the Pythagorean Theorem.



- 13.** The size of a TV set is described by the length of a diagonal of the screen. One TV is labelled 27 inches, which is about 70 cm. The screen is 40 cm high. What is the width of the screen? Give your answer to one decimal place. Draw a diagram to illustrate your answer.

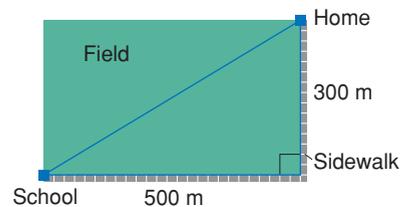


- 14. Assessment Focus** Look at the grid. Without measuring, find another point that is the same distance from A as B is. Explain your strategy. Show your work.



- 15.** A line segment joins points $P(-1, 2)$ and $Q(4, 5)$. Calculate the length of line segment PQ . Give your answer to one decimal place.

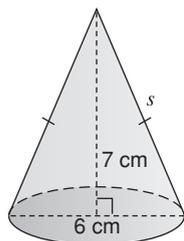
- 16.** Joanna usually uses the sidewalk to walk home from school. Today she is late, so she cuts through the field. How much shorter is Joanna's shortcut?



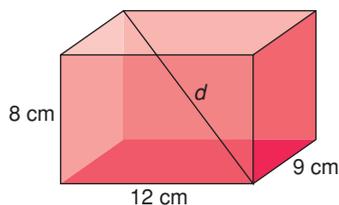
- 17.** Felix and Travis started at the same point in the campground. They walked in different directions. Felix walked 650 m and Travis walked 720 m. They were then 970 m apart. Were they travelling along paths that were at right angles to each other? Explain your thinking.
- 18.** A plane takes off from a local airport. It travels due north at a speed of 400 km/h. The wind blows the plane due east at a speed of 50 km/h. How far is the plane from the airport after 1 h? Give your answer to one decimal place.

- 19. Take It Further** A Powwow is a traditional practice in some First Nations cultures. Women dancers have small cone-shaped tin jingles sewn onto their dresses, one for each day of the year. A typical jingle has a triangular cross-section. Suppose the triangle has base 6 cm and height 7 cm.

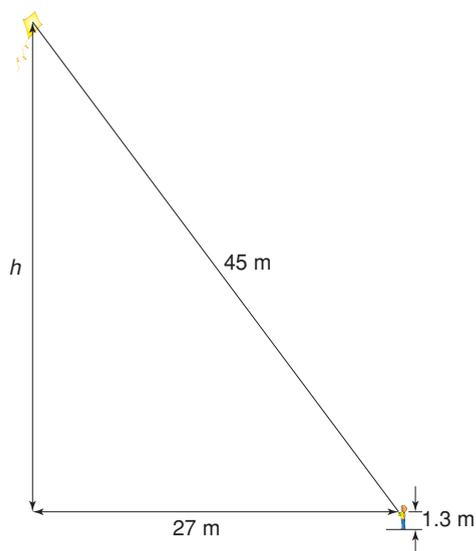
Use the diagram to help you find the slant height, s , of the jingle. Give your answer to one decimal place.



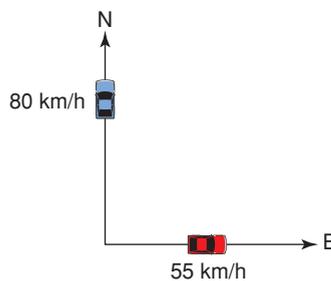
- 20. Take It Further** What is the length of the diagonal in this rectangular prism?



- 21. Take It Further** Rashad is flying a kite. His hand is 1.3 m above the ground. Use the picture below. How high is the kite above the ground?



- 22. Take It Further** Two cars meet at an intersection. One travels north at an average speed of 80 km/h. The other travels east at an average speed of 55 km/h. How far apart are the cars after 3 h? Give your answer to one decimal place.



Reflect

When can you use the Pythagorean Theorem to solve a problem? Use examples in your explanation.

Getting Unstuck

Have you ever got stuck trying to solve a math problem? Almost everybody has at one time or another.

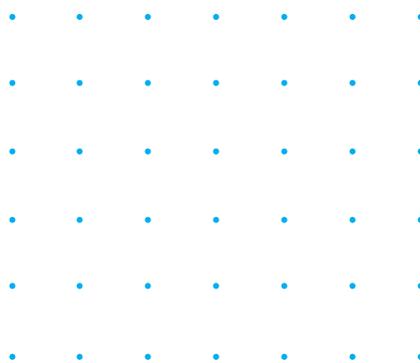
To be successful at math, you should know what to do when you get stuck trying to solve a problem.



Consider this problem:

Use square dot paper.

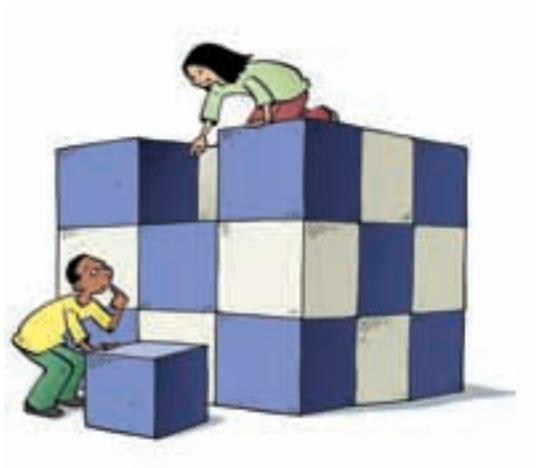
Draw a line segment with length $\sqrt{5}$ cm.



Suppose you get stuck.

Here are a few things you can try.

- Explain the problem to someone else in your own words.
- Make a simpler problem.
For example, how could you draw a line segment with length $\sqrt{2}$ cm?
- Draw a picture to represent the problem.
- Think about what you already know.
For example, what do you know about square roots?
How can you use dot paper to draw line segments of certain lengths?
- Try to approach the problem a different way.
- Talk to someone else about the problem.

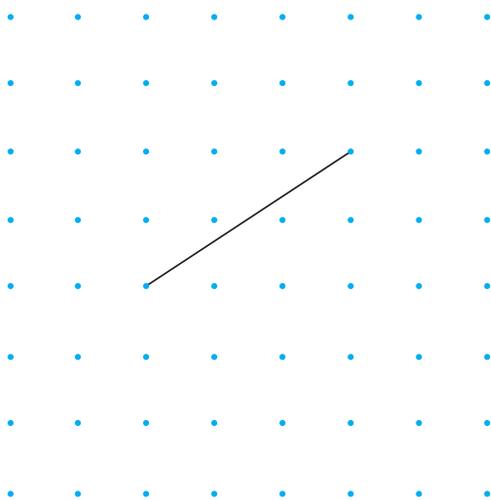


Using More Than One Strategy

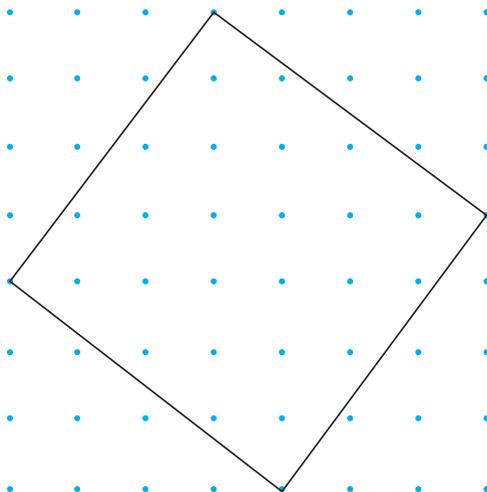
There is almost always more than one way to solve a problem. Often, the more ways you can use to solve a problem, the better your understanding of math and problem solving.

Use at least two different ways to show each statement below is true.

- $\sqrt{11}$ is between 3 and 4.
- This line segment has length $\sqrt{13}$ units.



- The number 25 is a perfect square.
- A triangle with side lengths 5 cm, 6 cm, and $\sqrt{11}$ cm is a right triangle.
- The area of this square is 25 square units.



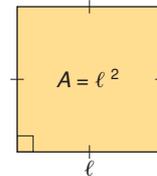
Unit Review

What Do I Need to Know?

✓ Side Length and Area of a Square

The side length of a square is equal to the square root of its area.

$$\text{Length} = \sqrt{\text{Area}}$$
$$\text{Area} = (\text{Length})^2$$



✓ The Approximate Square Root of a Number

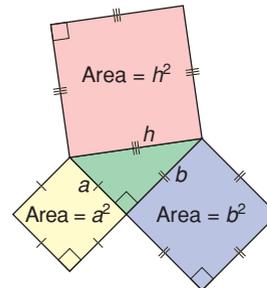
For numbers that are not perfect squares, we can determine the approximate square root using estimation or a calculator.

✓ The Pythagorean Theorem

In a right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the two legs.

$$h^2 = a^2 + b^2$$

Use the Pythagorean Theorem to find the length of a side in a right triangle, when two other sides are known.



What Should I Be Able to Do?

LESSON

1.1

1. Use square tiles.
Make as many different rectangles as you can with area 24 square units.
Draw your rectangles on grid paper.
Is 24 a perfect square?
Justify your answer.
2. Which of these numbers is a perfect square?
Use a diagram to support your answer.
a) 18 b) 25 c) 44 d) 80

LESSON

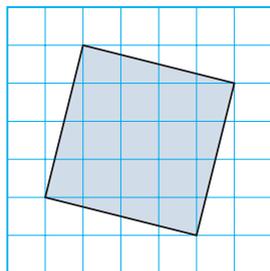
- 3.** I am a square number.
The sum of my digits is 7.
What square number might I be?
How many different numbers can you find?

1.2

- 4.** Find the square of each number.
a) 5 b) 7 c) 9 d) 13
- 5.** Find a square root.
a) 7^2 b) $\sqrt{289}$ c) $\sqrt{400}$
- 6. a)** List the factors of each number in ascending order.
i) 108 ii) 361 iii) 150
iv) 286 v) 324 vi) 56
b) Which numbers in part a are square numbers? How can you tell?
- 7.** The area of a square is 121 cm^2 .
What is the perimeter of the square?
How did you find out?

1.3

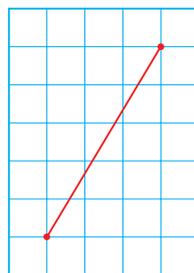
- 8.** Copy this square onto grid paper. Find its area. Then write the side length of the square.



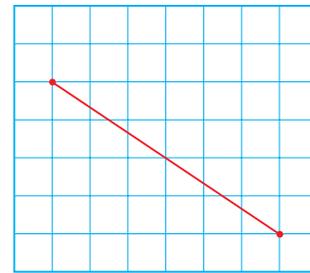
- 9.** The area of each square is given. Find its side length. Which side lengths are whole numbers?
a) 75 cm^2 b) 96 cm^2 c) 81 cm^2

- 10.** Without measuring, which line segment is longer? How can you tell?

a)



b)



- 11.** Find.
a) $\sqrt{26 \times 26}$
b) $\sqrt{5 \times 5}$
c) $\sqrt{50 \times 50}$
d) $\sqrt{13 \times 13}$

- 12.** Between which two consecutive whole numbers is each square root?
How did you find out?
a) $\sqrt{46}$ b) $\sqrt{84}$
c) $\sqrt{120}$ d) $\sqrt{1200}$

1.4

- 13.** Without using a calculator, estimate each square root to the nearest whole number.
a) $\sqrt{6}$ b) $\sqrt{11}$
c) $\sqrt{26}$ d) $\sqrt{35}$
e) $\sqrt{66}$ f) $\sqrt{86}$
-  **14.** Estimate each square root to one decimal place. Show your work.
a) $\sqrt{55}$ b) $\sqrt{75}$
c) $\sqrt{95}$ d) $\sqrt{105}$
e) $\sqrt{46}$ f) $\sqrt{114}$

15. Which is the better estimate of $\sqrt{72}$: 8.48 or 8.49? How do you know?

16. This First Nations quilt is a square, with area $16\,900\text{ cm}^2$. How long is each side of the quilt?



17. Is each statement true or false?

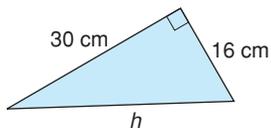
Justify your answers.

- a) $\sqrt{2} + \sqrt{2} = 2$
- b) $\sqrt{29}$ is between 5 and 6.
- c) $\sqrt{9} + \sqrt{25} = \sqrt{64}$

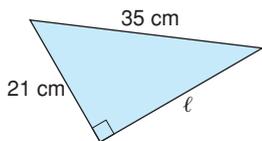
1.5

18. Find the length of each side labelled with a variable. Give your answers to one decimal place where needed.

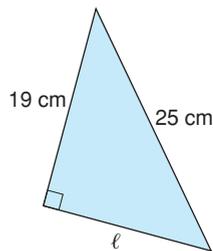
a)



b)

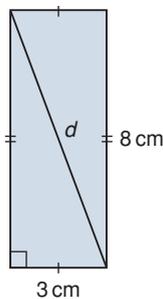


c)

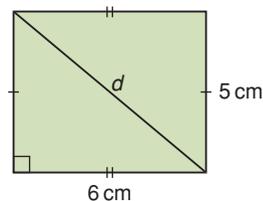


19. Find the length of the diagonal, d , in each rectangle. Give your answers to one decimal place where needed.

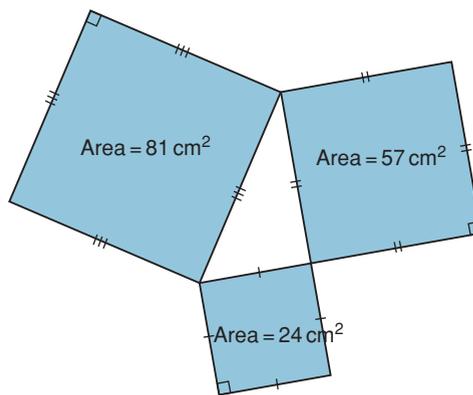
a)



b)



20. The area of the square on each side of a triangle is given. Is the triangle a right triangle? How do you know?



1.6

21. A triangle has side lengths 7 cm, 12 cm, and 15 cm. Is the triangle a right triangle? Justify your answer.

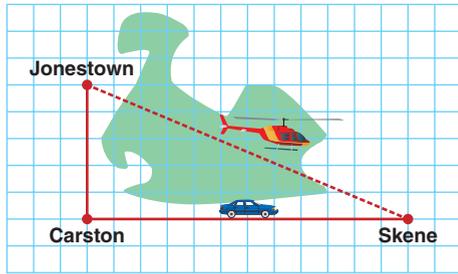
LESSON

- 22.** Identify the sets of numbers that are Pythagorean triples.
How did you decide?
- a) 24, 32, 40 b) 11, 15, 24
c) 25, 60, 65 d) 5, 8, 9

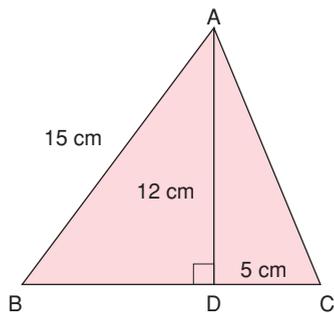
- 23.** Two numbers in a Pythagorean triple are 20 and 29.
Find the third number.
How many solutions are possible?
Justify your answer.

1.7

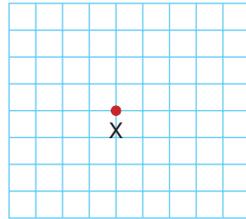
- 24.** Look at the map below. The side length of each grid square is 10 km.
How much farther is it to travel from Jonestown to Skene by car than by helicopter?



- 25.** Find the perimeter of $\triangle ABC$.

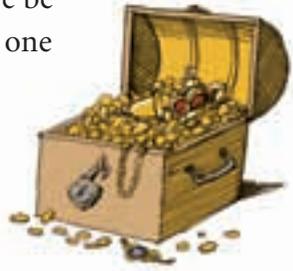


- 26.** There is a buried treasure at one of the points of intersection of the grid lines below.
Copy the grid.
Copy the grid.

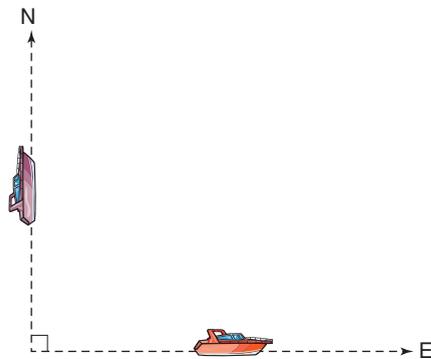


The treasure is $\sqrt{13}$ units from the point marked X.

- a) Where might the treasure be?
Explain how you located it.
b) Could there be more than one position?
Explain.



- 27.** Two boats started at the same point. After 2 h, one boat had travelled 20 km due east. The other boat had travelled 24 km due north. How far apart are the boats?
Explain your thinking.
Give your answer to one decimal place.



Practice Test

1. Find.

Give your answers to two decimal places where needed.

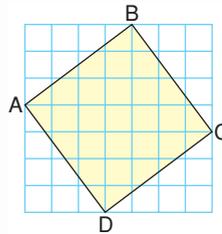
- a) $\sqrt{121}$ b) 14^2
c) $\sqrt{40}$ d) the square of 9

2. Explain why $\sqrt{1} = 1$.

3. A square tabletop has perimeter 32 cm.

What is the area of the tabletop? Explain your thinking.
Include a diagram.

4. a) What is the area of square ABCD?
b) What is the length of line segment AB?
Explain your reasoning.



5. The area of a square on each side of a triangle is given.

Is the triangle a right triangle?

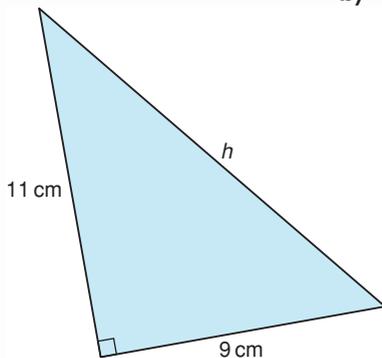
How do you know?

- a) 15 cm^2 , 9 cm^2 , 24 cm^2
b) 11 cm^2 , 7 cm^2 , 20 cm^2

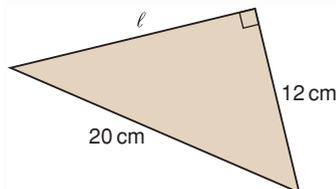
6. Find the length of each side labelled with a variable.

Give your answers to one decimal place where needed.

a)



b)



7. Which of the sets of numbers below is a Pythagorean triple?

How did you find out?

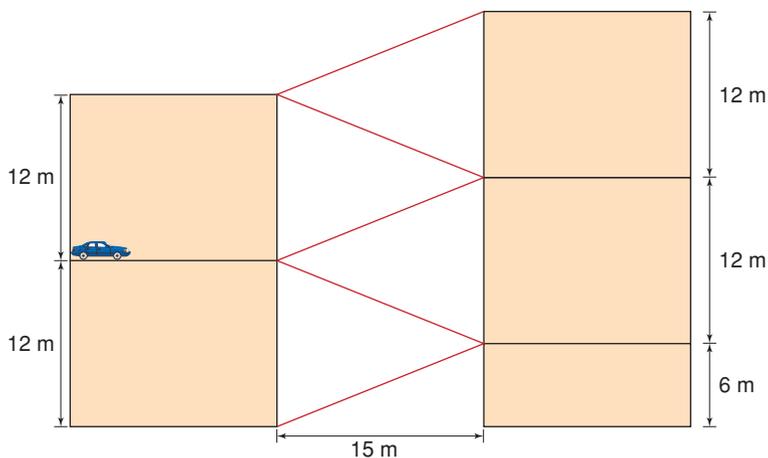
a) 20, 48, 54

b) 18, 24, 30

8. A parking garage in a shopping mall has ramps from one level to the next.

a) How long is each ramp?

b) What is the total length of the ramps?



9. Draw these 3 line segments on 1-cm grid paper.

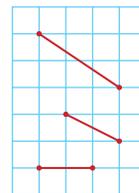
a) Find the length of each line segment to one decimal place.

b) Could these line segments be arranged to form a triangle?

If your answer is no, explain why not.

If your answer is yes, could they form a right triangle?

Explain.

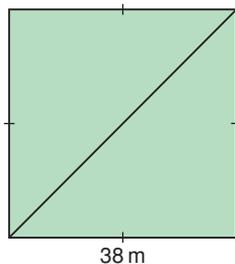


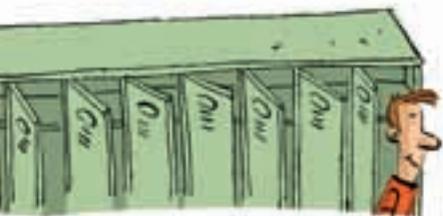
10. Rocco runs diagonally across a square field.

The side of the field has length 38 m.

How many times will Rocco have to run diagonally across the field to run a total distance of 1 km?

Give your answer to the nearest whole number.





Part A

In a school, there is a row of 25 lockers, numbered 1 to 25.

A student goes down the row and opens every locker.

A second student goes down the row and closes every other locker, beginning with locker 2.

A third student changes every 3rd locker, beginning with locker 3.

- If the locker is closed, the student opens it.
- If the locker is open, the student closes it.

A fourth student changes every 4th locker, beginning with locker 4.

This continues until 25 students have gone down the row.

Which lockers will be open after the 25th student has gone down the row?

1. Draw a chart similar to the one below.

Extend the chart for 25 lockers and 25 students.

The first student opens every locker.

Write O for each opened locker.

The second student closes every second locker, beginning with locker 2.

Write C for each locker that was closed.

The third student changes every third locker, beginning with locker 3. Write C or O for each locker that was changed.

Complete the chart for 25 students.

Locker \ Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
2		C		C		C		C		C		C		C	

2. Which lockers remain open after 25 students have gone down the row? What pattern do you notice?
How can you use this pattern to tell which lockers will be open?
3. Suppose there were 100 lockers and 100 students. Which lockers remain open after 100 students have gone down the row?

4. Suppose there were 400 lockers and 400 students. Which lockers remain open after 400 students have gone down the row?
5. What is the rule for any number of lockers and students? Why do you think your rule works?

Part B

6. Copy and complete a chart similar to the one below for the square numbers to 900. What patterns do you see?

Number	Square Numbers	Difference Between Square Numbers
1	1	3
2	4	5
3	9	7
4	16	9
5	25	

Check List

Your work should show:

- ✓ completed charts, clearly labelled
- ✓ clear descriptions of the patterns observed
- ✓ reasonable explanations of your thinking and your conclusions

7. We can use the completed chart to find some Pythagorean triples. For example, the difference between 25 (5^2) and 16 (4^2) is 9, or 3^2 . So, 3-4-5 is a Pythagorean triple.
 - a) How does it help to have the differences between square numbers in numerical order to identify a Pythagorean triple?
 - b) Will the difference between 2 consecutive square numbers ever be 16? Explain.
 - c) Which difference between square numbers will indicate the next Pythagorean triple?
 - d) Use your chart to identify 2 more Pythagorean triples.
 - e) Extend the chart to find 1 more Pythagorean triple.

Reflect on Your Learning

How do you think the Pythagorean Theorem could be used by a carpenter, a forester, a surveyor, a wildlife conservation officer, or the captain of a ship?